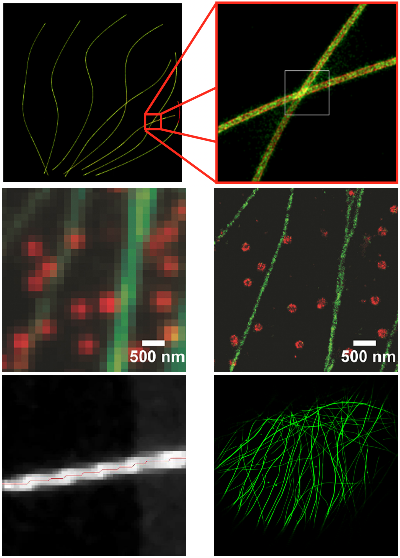
Motivations



Problème inverse

$$y = \mathbf{A}x$$

Exemple (Opérateur de dégradation)

- $\mathbf{A}$ = sous-échantillonnage
- $\mathbf{A}$ = flou
- ...

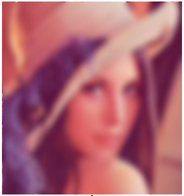
Problème inverse

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

Exemple (Opérateur de dégradation)

- $\mathbf{A}$ = sous-échantillonnage
- $\mathbf{A}$ = flou
- ...

$\mathbf{y}$



$\uparrow \mathbf{A}$

$\mathbf{x}$

Problème inverse

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

Exemple (Opérateur de dégradation)

- $\mathbf{A}$ = sous-échantillonnage
- $\mathbf{A}$ = flou
- ...

Problème inverse

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \epsilon$$

Exemple (Opérateur de dégradation)

- $\mathbf{A}$ = sous-échantillonnage
- $\mathbf{A}$ = flou
- ...

Problème inverse

$$\mathbf{y} - \mathbf{A}\mathbf{x} = \epsilon$$

Exemple (Opérateur de dégradation)

- $\mathbf{A}$ = sous-échantillonnage
- $\mathbf{A}$ = flou
- ...

Problème inverse

Minimisation (attache aux données)

Problème mal posé :

$$\arg\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|^2$$

Exemple (Opérateur de dégradation)

- $\mathbf{A}$ = sous-échantillonnage
- $\mathbf{A}$ = flou
- ...

Problème inverse

Minimisation (régularisation convexe)

$$\arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{A}\mathbf{x}\|^2 + \boxed{\lambda R(\mathbf{x})}$$

Exemple (Régulariseur)

- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_2^2$ (Tikhonov, 1963)
- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_1$ (Rudin et coll., 1992)
- $R(\mathbf{x}) = \|\mathbf{x}\|_{\mathcal{A}}$ (Chandrasekaran, 2010)

y

x

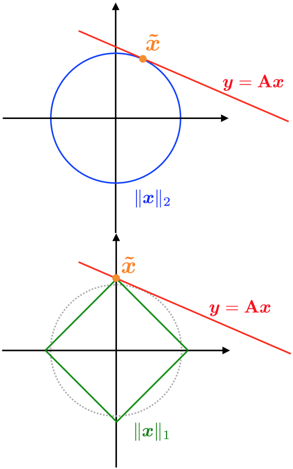
Problème inverse

Minimisation (régularisation convexe)

$$\arg\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|^2 + \boxed{\lambda R(\mathbf{x})}$$

Exemple (Régulariseur)

- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_2^2$ (Tikhonov, 1963)
- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_1$ (Rudin et coll., 1992)
- $R(\mathbf{x}) = \|\mathbf{x}\|_{\mathcal{A}}$ (Chandrasekaran, 2010)



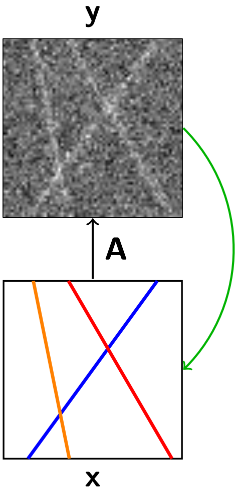
Problème inverse

Minimisation (régularisation convexe)

$$\arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|^2 + \boxed{\lambda R(\mathbf{x})}$$

Exemple (Régulariseur)

- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_2^2$ (Tikhonov, 1963)
- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_1$ (Rudin et coll., 1992)
- $R(\mathbf{x}) = \|\mathbf{x}\|_{\mathcal{A}}$ (Chandrasekaran, 2010)



Paradigme de la décomposition atomique

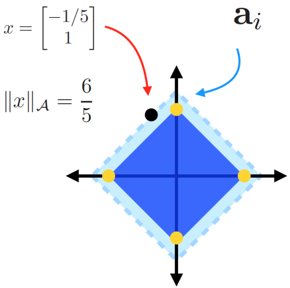
$$\mathbf{x} = \sum_{i=1}^{K} c_i \mathbf{a}_i, \quad c_i \geq 0, \quad \mathbf{a}_i \in \mathcal{A}$$

Norme atomique

$$\|\mathbf{x}\|_{\mathcal{A}} = \inf \{ t > 0 : \mathbf{x} \in t\,\mathrm{conv}(\mathcal{A}) \}$$

$$= \inf \left\{ \sum_{\mathbf{a} \in \mathcal{A}} c_{\mathbf{a}} : \mathbf{x} = \sum_{\mathbf{a} \in \mathcal{A}} c_{\mathbf{a}} \mathbf{a} \right\}$$

(Chandrasekaran et coll., 2010)



$$\mathcal{A} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}$$

$$\|\mathbf{x}\|_{\mathcal{A}} = \|\mathbf{x}\|_1$$

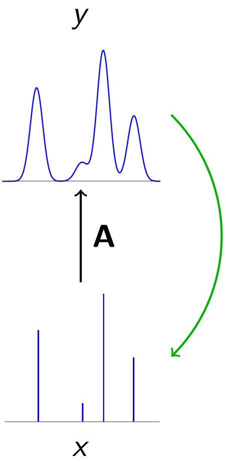
Application : super-résolution d'impulsions 1-D

$$x = \sum_{i=1}^{K} c_i \delta_{t_i}, \quad c_i \geq 0, \quad t_i \geq 0$$

Minimisation (régularisation convexe)

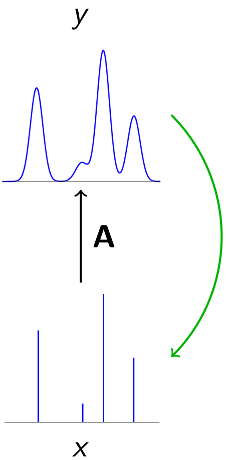
$$\arg\min_x \frac{1}{2} \|y - \mathbf{A}x\|^2 + \lambda \|x\|_{\mathrm{TV}}$$

Référence : (Candès et Fernandez-Granda, 2012)



Application : super-résolution d'impulsions 1-D

$$\mathcal{F}x = \sum_{i=1}^{K} c_i \mathrm{e}^{\mathrm{j}2\pi f_i \omega}, \quad c_i \geq 0, \quad t_i \geq 0$$



Minimisation (régularisation convexe)

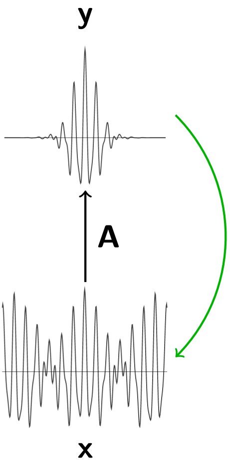
$$\arg\min_x \frac{1}{2} \|y - \mathbf{A}x\|^2 + \lambda \|\mathbf{x}\|_{\mathcal{A}}$$

Référence : (Tang, Bhaskar, Recht et coll., 2013)

Application : super-résolution d'impulsions 1-D

$$\mathbf{x} = \sum_{i=1}^{K} c_i \mathbf{a}(f_i), \quad c_i \geq 0, \quad \mathbf{a}(f_i) \in \mathcal{A}$$

$$\mathcal{A} = \left\{ \boldsymbol{a}(f) \in \mathbb{C}^N \right\}, \quad [\boldsymbol{a}(f)]_n = \mathrm{e}^{\mathrm{j}2\pi f n}$$



Minimisation (régularisation convexe)

$$\arg\min_{\mathbf{x}} \frac{1}{2} \left\|\mathbf{y} - \mathbf{Ax}\right\|^2 + \lambda \left\|\mathbf{x}\right\|_{\mathcal{A}}$$

Référence : (Tang, Bhaskar, Recht et coll., 2013)

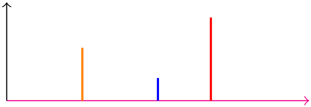
Problème inverse

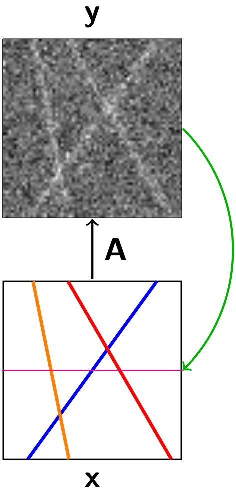
Minimisation (régularisation convexe)

$$\arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{A}\mathbf{x}\|^2 + \boxed{\lambda R(\mathbf{x})}$$

Exemple (Régulariseur)

- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_2^2$
- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_1$
- $R(\mathbf{x}) = \|\mathbf{x}\|_{\mathcal{A}}$ parcimonie





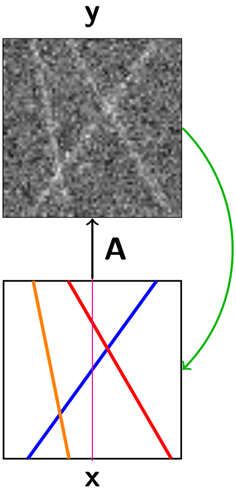
Problème inverse

Minimisation (régularisation convexe)

$$\arg\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|^2 + \boxed{\lambda R(\mathbf{x})}$$

Exemple (Régulariseur)

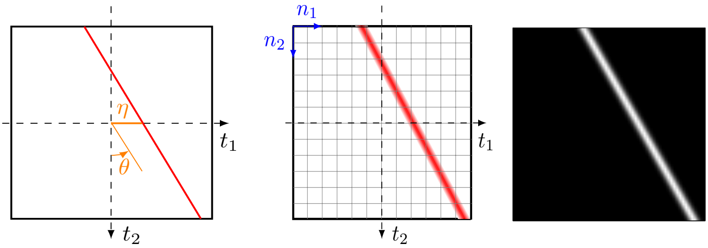
- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_2^2$
- $R(\mathbf{x}) = \|\nabla \mathbf{x}\|_1$
- $R(\mathbf{x}) = \|\mathbf{x}\|_{\mathcal{A}}$ parcimonie



Modélisation des droites

$$x^\sharp : (t_1, t_2) \in \mathbb{P} \mapsto \sum_{k=1}^{K} \alpha_k \delta\big(\cos\theta_k (t_1 - \eta_k) + \sin\theta_k \, t_2\big)$$

$$\mathbf{b}^\sharp[n_1, n_2] = (x^\sharp * \phi)(n_1, n_2)$$



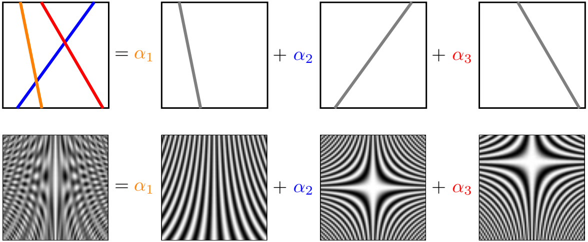
$x^\sharp$ $\quad$ $x^\sharp * \phi$ $\quad$ $\mathbf{b}^\sharp$

Modélisation des droites

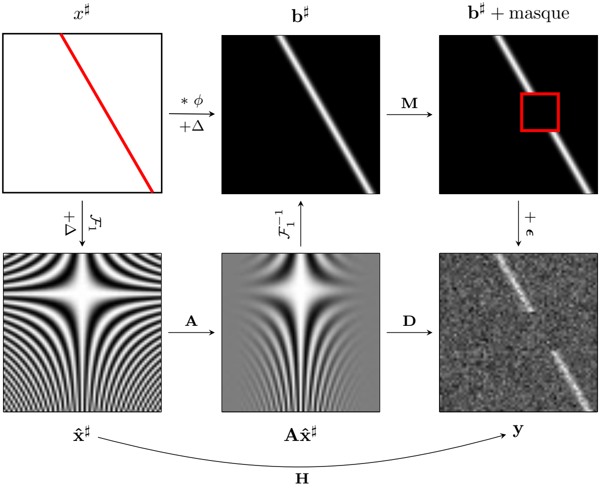
$$\hat{\mathbf{x}}^\sharp[m,n_2] = (\mathcal{F}_1 x^\sharp)[m,n_2] = \sum_{k=1}^{K} c_k \mathrm{e}^{\mathrm{j}2\pi\left(\frac{\tan\theta_k}{W} n_2 + \frac{\eta_k}{W}\right) m}$$

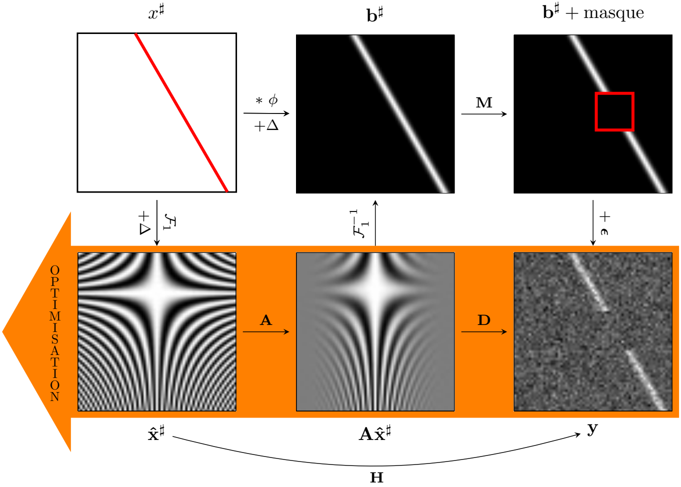
$$c_k = \frac{\alpha_k}{\cos\theta_k} \geq 0$$

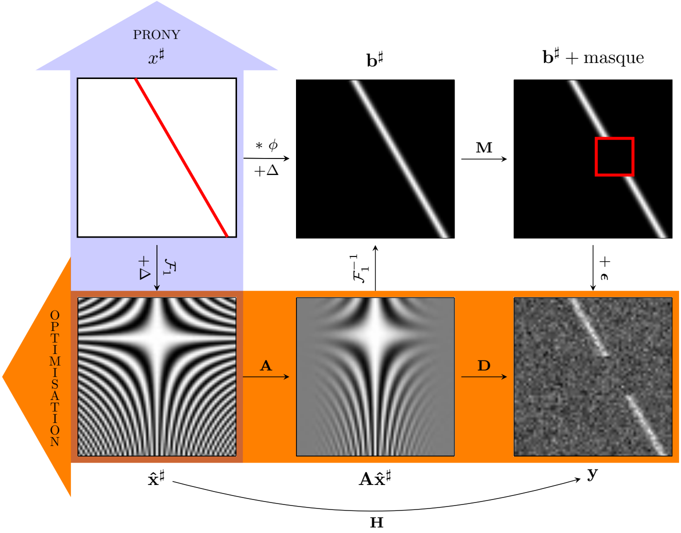
$$\hat{\mathbf{b}}^\sharp[m,:] = (\hat{g}[m]\hat{\mathbf{x}}^\sharp[m,:]) * \boldsymbol{h} \rightarrow \mathbf{A}\hat{\mathbf{x}}^\sharp = \hat{\mathbf{b}}^\sharp$$



Étapes de reconstruction




$x^\sharp$
$\mathbf{b}^\sharp$
$\mathbf{b}^\sharp$ + masque
$* \phi$
$+\Delta$
$\mathbf{M}$
$+\Delta$
$\mathcal{F}_1$
$\mathcal{F}_1^{-1}$
$+\epsilon$
OPTIMISATION
$\mathbf{A}$
$\mathbf{D}$
$\hat{\mathbf{x}}^\sharp$
$\mathbf{A}\hat{\mathbf{x}}^\sharp$
$\mathbf{y}$
$\mathbf{H}$


PRONY
$x^\sharp$
$\mathbf{b}^\sharp$
$\mathbf{b}^\sharp$ + masque
$*\ \phi$
$+\Delta$
$\mathbf{M}$
$+\Delta$
$\mathcal{F}_1$
$\mathcal{F}_1^{-1}$
$+\,\epsilon$
OPTIMISATION
$\mathbf{A}$
$\mathbf{D}$
$\hat{\mathbf{x}}^\sharp$
$\mathbf{A}\hat{\mathbf{x}}^\sharp$
$\mathbf{y}$
$\mathbf{H}$

Décomposition atomique des lignes

$$\hat{\mathbf{x}}^{\sharp}[m, n_2] = \sum_{k=1}^{K} c_k \mathrm{e}^{\mathrm{j}2\pi\left(\frac{\tan\theta_k}{W} n_2 + \frac{\eta_k}{W}\right)m}$$

$$\boldsymbol{l}^{\sharp}_{n_2} = \hat{\mathbf{x}}^{\sharp}[:, n_2] = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{n_2,k}, 0), \quad [\boldsymbol{a}(f, \phi)]_i = \mathrm{e}^{\mathrm{j}(2\pi f i + \phi)} \in \mathcal{A}$$

Décomposition atomique des colonnes

$$\hat{\mathbf{x}}^\sharp[m, n_2] = \sum_{k=1}^{K} c_k \mathrm{e}^{\mathrm{j}2\pi\left(\frac{\tan\theta_k}{W} m\right) n_2 + \frac{2\pi\eta_k m}{W}}$$

$$\boldsymbol{t}_m^\sharp = \hat{\mathbf{x}}^\sharp[m, :] = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{m,k}, \phi_{m,k})^\mathsf{T}, \quad [\boldsymbol{a}(f, \phi)]_i = \mathrm{e}^{\mathrm{j}(2\pi f i + \phi)} \in \mathcal{A}$$

Décomposition atomique des lignes et colonnes

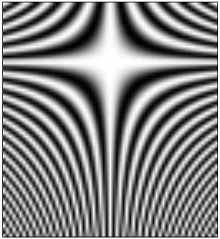
$$\hat{\mathbf{x}}^\sharp[m, n_2] = \sum_{k=1}^{K} c_k \mathrm{e}^{\mathrm{j}2\pi\left(\frac{\tan\theta_k}{W} n_2 + \frac{\eta_k}{W}\right)m}$$

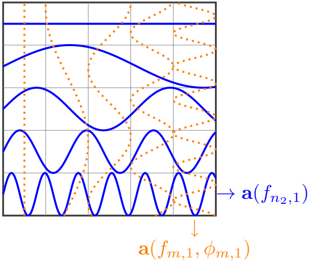
1. $\boldsymbol{l}^\sharp_{n_2} = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{n_2,k}, 0)$ (lignes de $\hat{\mathbf{x}}$, sans phase)
2. $\boldsymbol{t}^\sharp_{m} = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{m,k}, \phi_{m,k})^{\mathsf{T}}$ (colonnes de $\hat{\mathbf{x}}$, avec phase)

Décomposition atomique d'une droite ($K = 1$)

$$\hat{\mathbf{x}}^{\sharp}[m, n_2] = c_1 \mathrm{e}^{\mathrm{j}2\pi\left(\frac{\tan\theta_1}{W} n_2 + \frac{\eta_1}{W}\right)m}$$

1. $\boldsymbol{l}^{\sharp}_{n_2} = c_1 \boldsymbol{a}(f_{n_2,1}, 0)$ (un atome sans phase)
2. $\boldsymbol{t}^{\sharp}_{m} = c_1 \boldsymbol{a}(f_{m,1}, \phi_{m,1})^{\mathsf{T}}$ (un atome avec phase)





Normes atomiques

$$\hat{\mathbf{x}}^\sharp[m, n_2] = \sum_{k=1}^{K} c_k \mathrm{e}^{\mathrm{j}2\pi\left(\frac{\tan\theta_k}{W} n_2 + \frac{\eta_k}{W}\right)m}, \quad c^\star = \sum_{k=1}^{K} c_k$$

1. $\boldsymbol{l}_{n_2}^\sharp = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{n_2,k}, 0)$ (lignes de $\hat{\mathbf{x}}$, sans phase)
2. $\boldsymbol{t}_m^\sharp = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{m,k}, \phi_{m,k})^\mathsf{T}$ (colonnes de $\hat{\mathbf{x}}$, avec phase)

Norme atomique :

$$\|\boldsymbol{z}\|_{\mathcal{A}} = \inf_{c'_k, f'_k, \phi'_k} \left\{ \sum_k c'_k : \boldsymbol{z} = \sum_k c'_k \boldsymbol{a}(f'_k, \phi'_k) \right\}$$

Normes atomiques

1. $\boldsymbol{l}^{\sharp}_{n_2} = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{n_2,k}, 0)$

$\hookrightarrow \mathbf{T}_{M+1}(\boldsymbol{l}^{\sharp}_{n_2}) \succcurlyeq 0$ + de rang K (Carathéodory, 1907)

$\hookrightarrow \|\boldsymbol{l}^{\sharp}_{n_2}\|_{\mathcal{A}} = \sum_{k=1}^{K} c_k = \hat{\mathbf{x}}^{\sharp}[0, n_2]$

2. $\boldsymbol{t}^{\sharp}_{m} = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{m,k}, \phi_{m,k})^{\top}$ (Tang et coll., 2013)

$$\|\boldsymbol{t}^{\sharp}_{m}\|_{\mathcal{A}} = \inf_{\boldsymbol{q}\in\mathbb{C}^N, t\in\mathbb{R}} \left\{ \frac{1}{2}\mathrm{Tr}(\mathbf{T}_N(\boldsymbol{q})) + \frac{1}{2}t : \begin{pmatrix} \mathbf{T}_N(\boldsymbol{q}) & \boldsymbol{t}^{\sharp}_{m} \\ \boldsymbol{t}^{\sharp\,*}_{m} & t \end{pmatrix} \succcurlyeq 0 \right\} .$$

Normes atomiques

1. $\boldsymbol{l}^\sharp_{n_2} = \sum_{k=1}^K c_k \boldsymbol{a}(f_{n_2,k}, 0)$

$\hookrightarrow \mathbf{T}_{M+1}(\boldsymbol{l}^\sharp_{n_2}) \succcurlyeq 0$ + de rang K (Carathéodory, 1907)

$\hookrightarrow \|\boldsymbol{l}^\sharp_{n_2}\|_{\mathcal{A}} = \sum_{k=1}^K c_k = \hat{\mathbf{x}}^\sharp[0, n_2] = c^\star$

2. $\boldsymbol{t}^\sharp_m = \sum_{k=1}^K c_k \boldsymbol{a}(f_{m,k}, \phi_{m,k})^{\mathsf{T}}$ (Polisano et coll., 2016)

$$\|\boldsymbol{t}^\sharp_m\|_{\mathcal{A}} = \min_{\boldsymbol{q}\in\mathbb{C}^N} \left\{ q_0 : \underbrace{\begin{pmatrix} \mathbf{T}_N(\boldsymbol{q}) & \boldsymbol{t}^\sharp_m \\ {\boldsymbol{t}^\sharp_m}^* & q_0 \end{pmatrix}}_{\mathbf{T}'_N(\boldsymbol{t}^\sharp_m, \boldsymbol{q})} \succcurlyeq 0 \right\} \equiv \mathrm{SDP}(\boldsymbol{t}^\sharp_m),$$

$\hookrightarrow \|\boldsymbol{t}^\sharp_m\|_{\mathcal{A}} = \mathrm{SDP}(\boldsymbol{t}^\sharp_m) = \boldsymbol{q}_m[0] \leqslant c^\star$

Normes atomiques

$$\hat{\mathbf{x}}^\sharp[m, n_2] = \sum_{k=1}^{K} c_k \mathrm{e}^{\mathrm{j}2\pi\left(\frac{\tan\theta_k}{W} n_2 + \frac{\eta_k}{W}\right)m}, \quad c^\star = \sum_{k=1}^{K} c_k$$

1. $\boldsymbol{l}_{n_2}^\sharp = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{n_2,k}, 0)$ (lignes de $\hat{\mathbf{x}}$, sans phase)
2. $\boldsymbol{t}_m^\sharp = \sum_{k=1}^{K} c_k \boldsymbol{a}(f_{m,k}, \phi_{m,k})^\mathsf{T}$ (colonnes de $\hat{\mathbf{x}}$, avec phase)

Caractérisation (convexe) des K droites par la norme atomique

1. $\|\boldsymbol{l}_{n_2}^\sharp\|_\mathcal{A} = c^\star = \hat{\mathbf{x}}^\sharp[0, n_2]$ et $\mathbf{T}_{M+1}(\boldsymbol{l}_{n_2}^\sharp) \succcurlyeq 0$
2. $\|\boldsymbol{t}_m^\sharp\|_\mathcal{A} = \mathrm{SDP}(\boldsymbol{t}_m^\sharp) = \boldsymbol{q}_m[0] \leqslant c^\star$, $\mathbf{T}'_{H_S}(\boldsymbol{t}_m^\sharp, \boldsymbol{q}_m) \succcurlyeq 0$

Problème d'optimisation convexe

Proposition (Minimisation convexe)

$$\tilde{\mathbf{x}} \in \underset{\hat{\mathbf{x}},\mathbf{q} \in \mathcal{X} \times \mathcal{Q}}{\arg\min} \frac{1}{2} \|\mathbf{A}\hat{\mathbf{x}} - \hat{\mathbf{y}}\|^2 \ ,$$

$$\text{sous contraintes} \quad \begin{cases} \forall n_2 = 0, \dots, H_S - 1, \ \forall m = 0, \dots, M \ , \\ \hat{\mathbf{x}}[0, n_2] = \hat{\mathbf{x}}[0, 0] \leqslant c \ , \\ \mathbf{q}[m, 0] \leqslant c \ , \\ \mathbf{T}'_{H_S}(\hat{\mathbf{x}}[m, :], \mathbf{q}[m, :]) \succcurlyeq 0 \ , \\ \mathbf{T}_{M+1}(\hat{\mathbf{x}}[:, n_2]) \succcurlyeq 0 \ . \end{cases}$$

Problème d'optimisation convexe

Proposition (Minimisation convexe)

$$\tilde{\mathbf{x}} \in \underset{\hat{\mathbf{x}}, \mathbf{q} \in \mathcal{X} \times \mathcal{Q}}{\arg\min} \frac{1}{2} \|\mathbf{A}\hat{\mathbf{x}} - \hat{\mathbf{y}}\|^2 ,$$

$$\text{sous contraintes} \quad \begin{cases} \forall n_2 = 0, \ldots, H_S - 1, \ \forall m = 0, \ldots, M , \\ \hat{\mathbf{x}}[0, n_2] = \hat{\mathbf{x}}[0, 0] \leqslant c , \\ \mathbf{q}[m, 0] \leqslant c , \\ \mathbf{T}'_{H_S}(\hat{\mathbf{x}}[m, :], \mathbf{q}[m, :]) \succcurlyeq 0 , \\ \mathbf{T}_{M+1}(\hat{\mathbf{x}}[:, n_2]) \succcurlyeq 0 . \end{cases}$$

(Chambolle et Pock, 2010)

$$\tilde{\mathbf{X}} = \underset{\mathbf{X} \in \mathcal{H}}{\arg\min} \left\{ F(\mathbf{X}) + G(\mathbf{X}) + \sum_{i=0}^{Q-1} H_i(\mathrm{L}_i(\mathbf{X})) \right\}$$

Problème d'optimisation convexe

Proposition (Minimisation convexe)

$$\tilde{\mathbf{x}} \in \underset{\hat{\mathbf{x}},\mathbf{q} \in \mathcal{X} \times \mathcal{Q}}{\arg\min} \frac{1}{2} \|\mathbf{A}\hat{\mathbf{x}} - \hat{\mathbf{y}}\|^2 ,$$

$$\text{sous contraintes} \quad \begin{cases} \forall n_2 = 0, \ldots, H_S - 1, \ \forall m = 0, \ldots, M , \\ \hat{\mathbf{x}}[0, n_2] = \hat{\mathbf{x}}[0, 0] \leqslant c , \\ \mathbf{q}[m, 0] \leqslant c , \\ \mathbf{T}'_{H_S}(\hat{\mathbf{x}}[m, :], \mathbf{q}[m, :]) \succcurlyeq 0 , \\ \mathbf{T}_{M+1}(\hat{\mathbf{x}}[:, n_2]) \succcurlyeq 0 . \end{cases}$$

(Chambolle et Pock, 2010)

$$\tilde{\mathbf{X}} = \underset{\mathbf{X} \in \mathcal{H}}{\arg\min} \left\{ F(\mathbf{X}) + G(\mathbf{X}) + \sum_{i=0}^{Q-1} H_i(\mathrm{L}_i(\mathbf{X})) \right\}$$

Problème d'optimisation convexe

Proposition (Minimisation convexe)

$$\tilde{\mathbf{x}} \in \underset{\hat{\mathbf{x}},\mathbf{q} \in \mathcal{X}\times\mathcal{Q}}{\arg\min} \frac{1}{2}\|\mathbf{A}\hat{\mathbf{x}} - \hat{\mathbf{y}}\|^2 ,$$

$$\text{sous contraintes} \quad \begin{cases} \forall n_2 = 0, \dots, H_S - 1, \ \forall m = 0, \dots, M , \\ \hat{\mathbf{x}}[0, n_2] = \hat{\mathbf{x}}[0,0] \leqslant c , \\ \mathbf{q}[m, 0] \leqslant c , \\ \mathbf{T}'_{H_S}(\hat{\mathbf{x}}[m,:], \mathbf{q}[m,:]) \succcurlyeq 0 , \\ \mathbf{T}_{M+1}(\hat{\mathbf{x}}[:, n_2]) \succcurlyeq 0 . \end{cases}$$

(Chambolle et Pock, 2010)

$$\tilde{\mathbf{X}} = \underset{\mathbf{X}\in\mathcal{H}}{\arg\min} \left\{ F(\mathbf{X}) + G(\mathbf{X}) + \sum_{i=0}^{Q-1} H_i(\mathrm{L}_i(\mathbf{X})) \right\}$$

Problème d'optimisation convexe

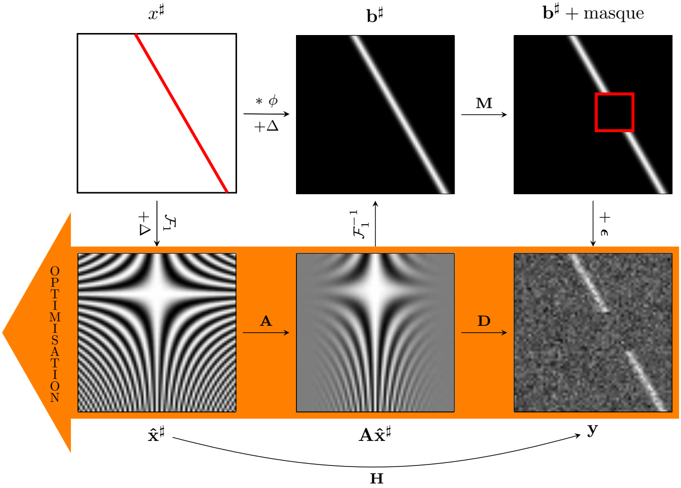
Proposition (Minimisation convexe)

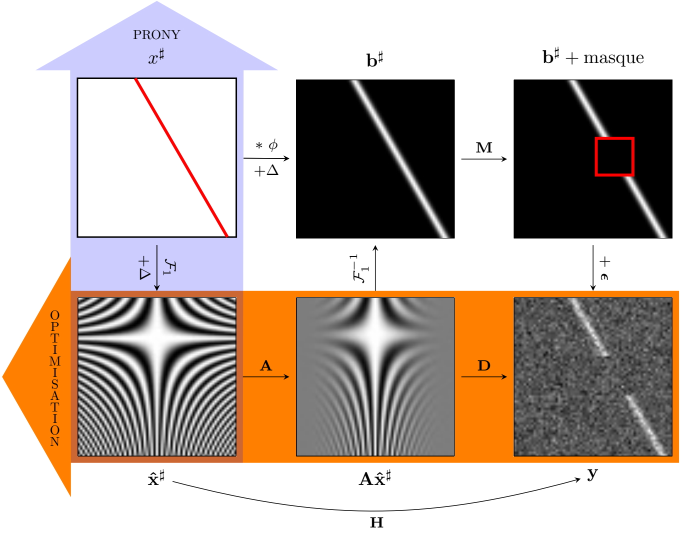
$$\tilde{\mathbf{x}} \in \underset{\hat{\mathbf{x}},\mathbf{q} \in \mathcal{X}\times\mathcal{Q}}{\arg\min} \frac{1}{2}\|\mathbf{A}\hat{\mathbf{x}} - \hat{\mathbf{y}}\|^2 ,$$

$$\text{sous contraintes} \quad \begin{cases} \forall n_2 = 0, \ldots, H_S - 1, \ \forall m = 0, \ldots, M , \\ \hat{\mathbf{x}}[0, n_2] = \hat{\mathbf{x}}[0, 0] \leqslant c , \\ \mathbf{q}[m, 0] \leqslant c , \\ \mathbf{T}'_{H_S}(\hat{\mathbf{x}}[m, :], \mathbf{q}[m, :]) \succcurlyeq 0 , \\ \mathbf{T}_{M+1}(\hat{\mathbf{x}}[:, n_2]) \succcurlyeq 0 . \end{cases}$$

(Chambolle et Pock, 2010)

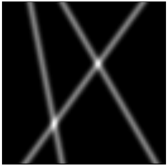
$$\tilde{\mathbf{X}} = \underset{\mathbf{X}\in\mathcal{H}}{\arg\min} \left\{ F(\mathbf{X}) + G(\mathbf{X}) + \sum_{i=0}^{Q-1} H_i(\mathrm{L}_i(\mathbf{X})) \right\}$$


$x^\sharp$
$\mathbf{b}^\sharp$
$\mathbf{b}^\sharp$ + masque
$* \, \phi$
$+\Delta$
$\mathbf{M}$
$+\Delta$
$\mathcal{F}_1$
$\mathcal{F}_1^{-1}$
$+\epsilon$
OPTIMISATION
$\mathbf{A}$
$\mathbf{D}$
$\hat{\mathbf{x}}^\sharp$
$\mathbf{A}\hat{\mathbf{x}}^\sharp$
$\mathbf{y}$
$\mathbf{H}$

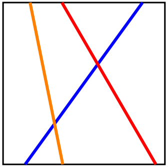

PRONY
$x^\sharp$
$\mathbf{b}^\sharp$
$\mathbf{b}^\sharp$ + masque
$*\ \phi$
$+\Delta$
$\mathbf{M}$
$\mathcal{F}_1$
$+\Delta$
$\mathcal{F}_1^{-1}$
$+\epsilon$
OPTIMISATION
$\mathbf{A}$
$\mathbf{D}$
$\hat{\mathbf{x}}^\sharp$
$\mathbf{A}\hat{\mathbf{x}}^\sharp$
$\mathbf{y}$
$\mathbf{H}$

Expériences numériques

- Débruitage et déconvolution



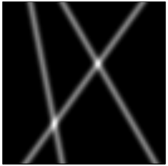
Exp. 1



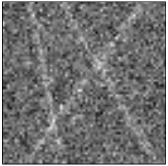
Détection

Expériences numériques

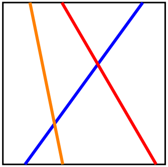
- Débruitage et déconvolution



Exp. 1



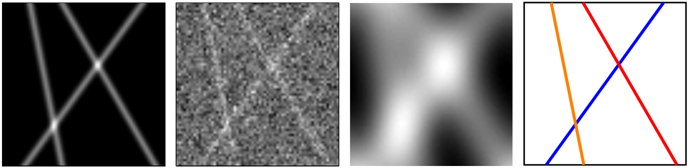
Exp. 2



Détection

Expériences numériques

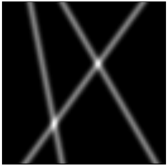
- Débruitage et déconvolution



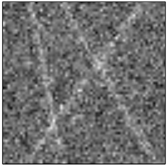
Exp. 1 | Exp. 2 | Exp 3. | Détection

Expériences numériques

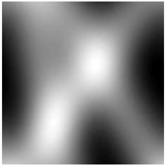
- Débruitage et déconvolution



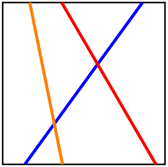
Exp. 1



Exp. 2



Exp 3.



Détection

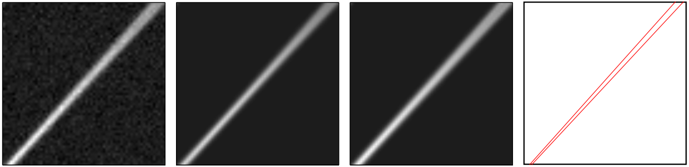
TABLE: Erreurs relatives de l'estimation des paramètres des lignes

	Expérience 1	**Expérience 2**	**Expérience 3**
Δ_θ/θ	$(10^{-7}, 3.10^{-6}, 7.10^{-7})$	$(10^{-2}, 6.10^{-2}, 9.10^{-2})$	$(6.10^{-7}, 9.10^{-5}, 8.10^{-6})$
Δ_α/α	$(10^{-7}, 10^{-7}, 10^{-7})$	$(10^{-2}, 9.10^{-2}, 2.10^{-1})$	$(4.10^{-5}, 2.10^{-5}, 2.10^{-5})$
Δ_η	$(4.10^{-6}, 7.10^{-6}, 7.10^{-6})$	$(5.10^{-2}, 4.10^{-2}, 3.10^{-2})$	$(5.10^{-5}, 10^{-4}, 3.10^{-4})$

Expériences numériques

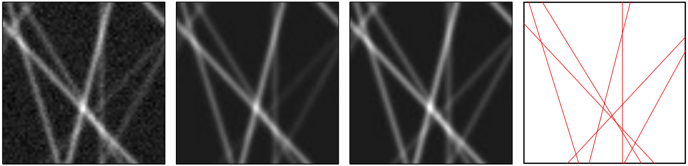
Expériences numériques

- Lignes proches



Bruitée | Débruitée | Sans bruit | Détection

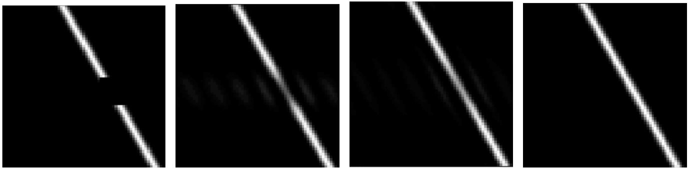
- Lignes multiples



Bruitée | Débruitée | Sans bruit | Détection

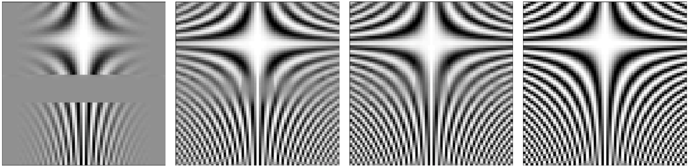
Expériences numériques

- Inpainting spatial



Masquage | iter $= 2000$ | iter $= 10000$ | iter $\to \infty$

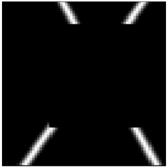
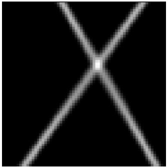
- Inpainting en Fourier

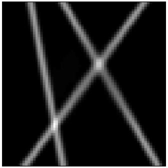


Masquage | iter $= 2000$ | iter $= 10000$ | iter $\to \infty$

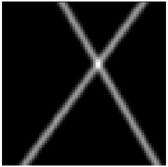
Expériences numériques

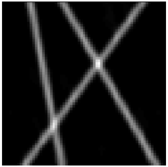
- Inpainting masquage important


Masquage

Inpainting

Masquage

Inpainting

- Inpainting masquage aléatoire


Masquage

Inpainting

Masquage

Inpainting