TEXTURE MODELING BY GAUSSIAN FIELDS WITH PRESCRIBED LOCAL ORIENTATION

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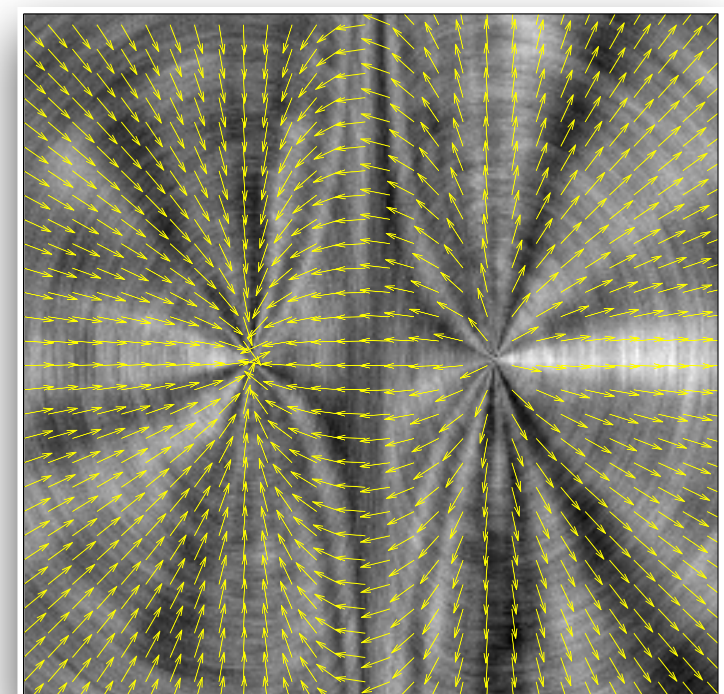
joint work with

Marianne Clausel
Valérie Perrier
Laurent Condat



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Outline

- Introduction
  - Motivation
  - General probabilistic framework
- Our new stochastic model
  - Definition: Locally Anisotropic Fractional Brownian Field
  - Notion of tangent field
- Synthesis methods
  - Tangent field simulation by a turning bands method
  - LAFBF simulation via tangent field formulation
- Numerical experiments
- Conclusion and future work

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Mathematical
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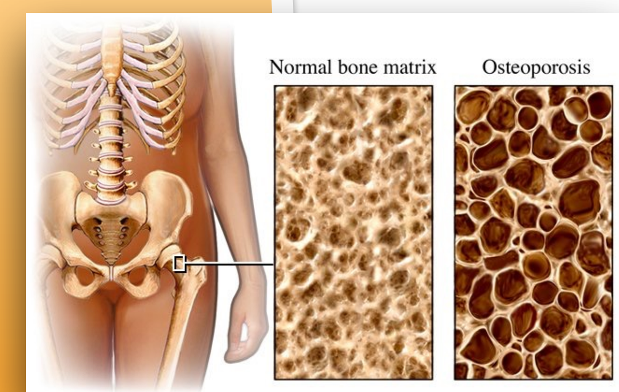

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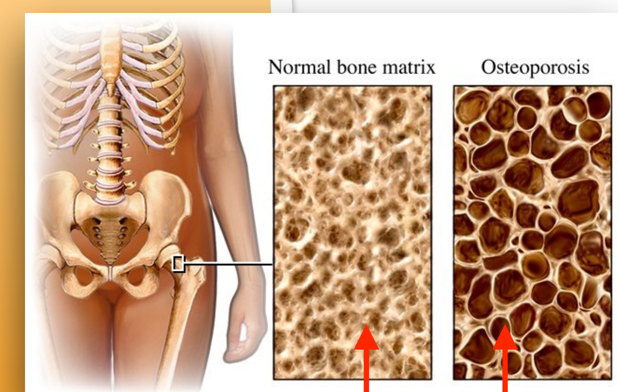

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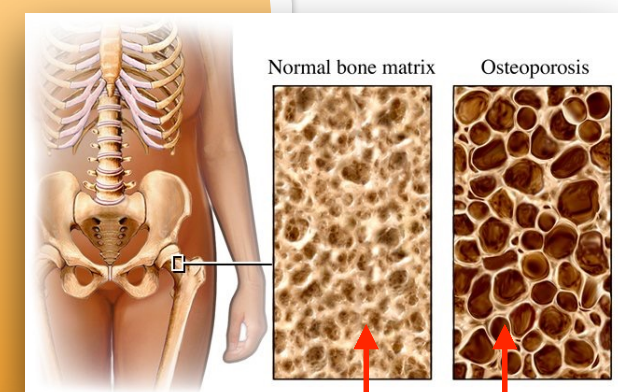

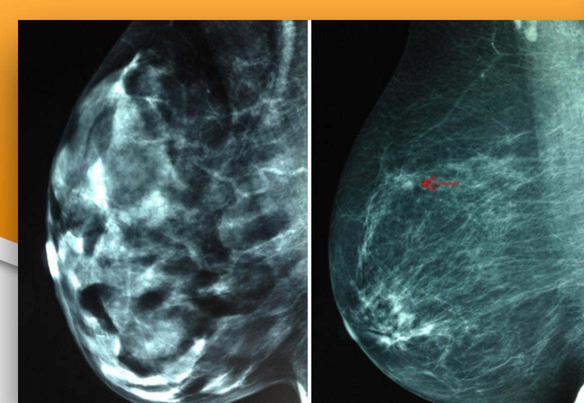

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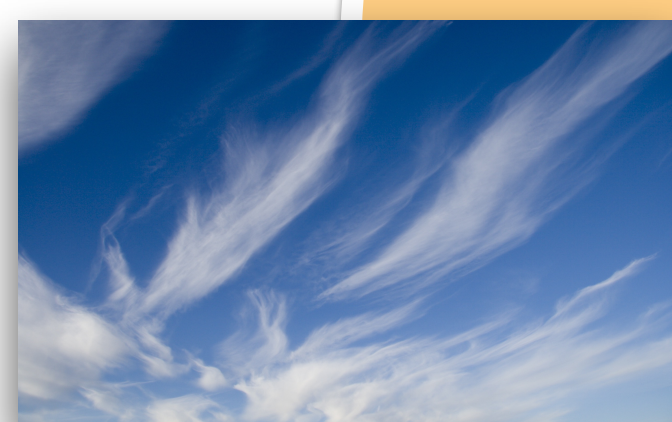

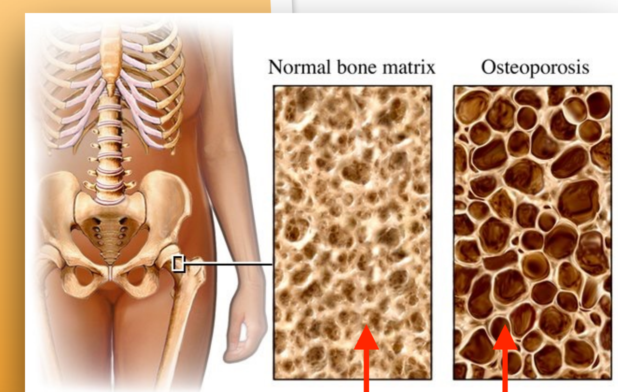

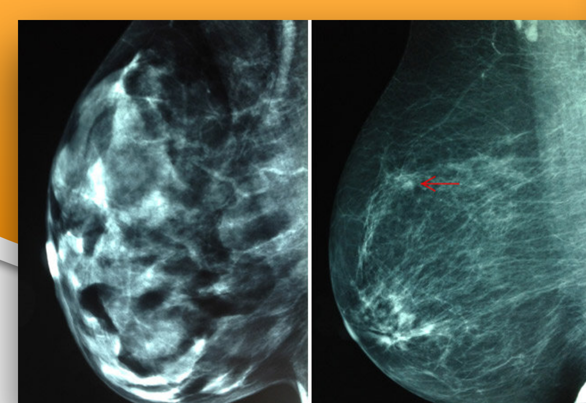

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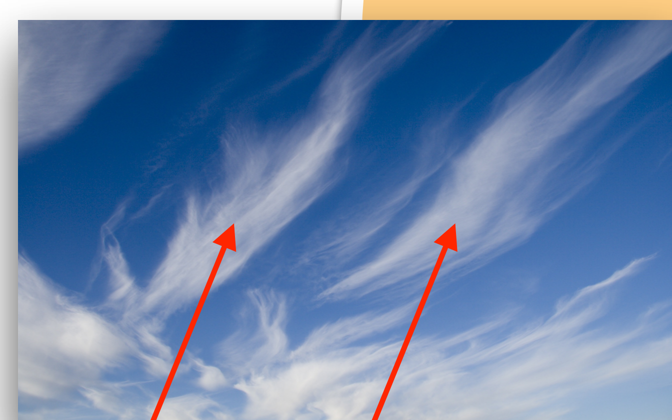

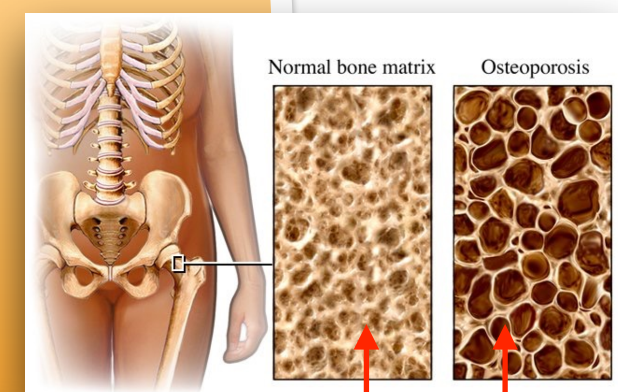

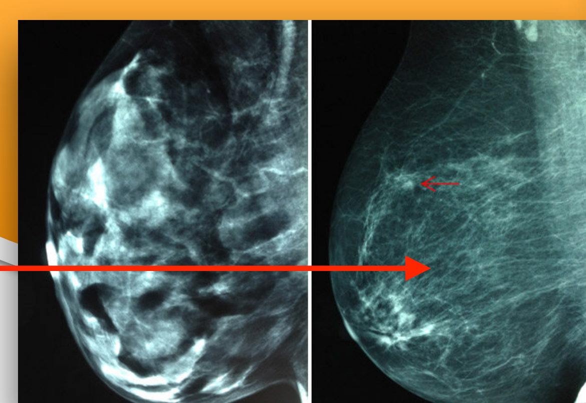

Orientation and anisotropy

Roughness and regularity

The basic component : Fractional Brownian Field (FBF)

- B^H FBF with Hurst index $0 < H < 1$ [Mandelbrot,Van Ness,1968]
  - stationary increments : $B^H(\cdot + \mathbf{z}) - B^H(\mathbf{z}) \stackrel{\mathcal{L}}{=} B^H(\cdot) - B^H(0)$
  - self-similar : $B^H(\lambda \cdot) \stackrel{\mathcal{L}}{=} \lambda^H B^H(\cdot)$
  - isotropic : $B^H \circ R_\theta \stackrel{\mathcal{L}}{=} B^H$
- The covariance is given by

$$\mathrm{Cov}(B^H(\mathbf{x}), B^H(\mathbf{y})) = c_H(\|\mathbf{x}\|^{2H} + \|\mathbf{y}\|^{2H} - \|\mathbf{x} - \mathbf{y}\|^{2H})$$

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- Harmonizable representation [Samorodnitsky,Taqqu,1997]

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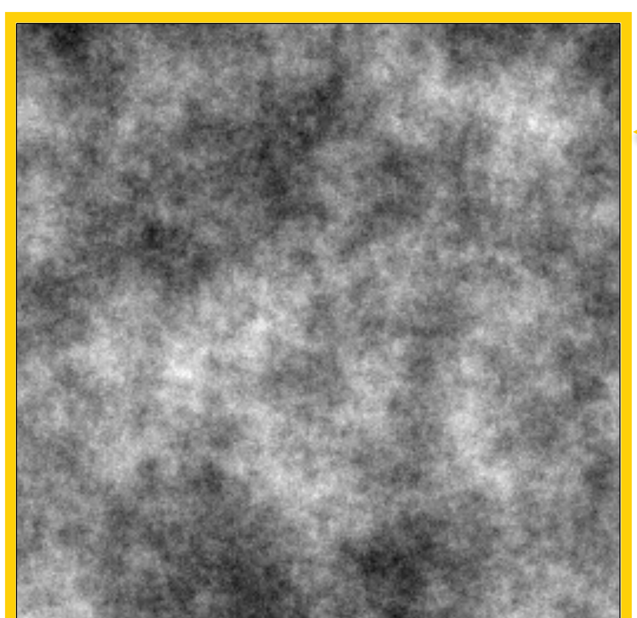

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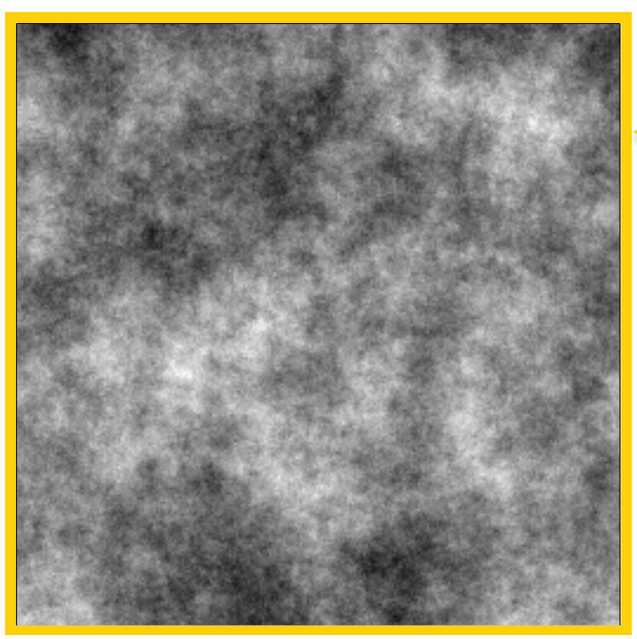
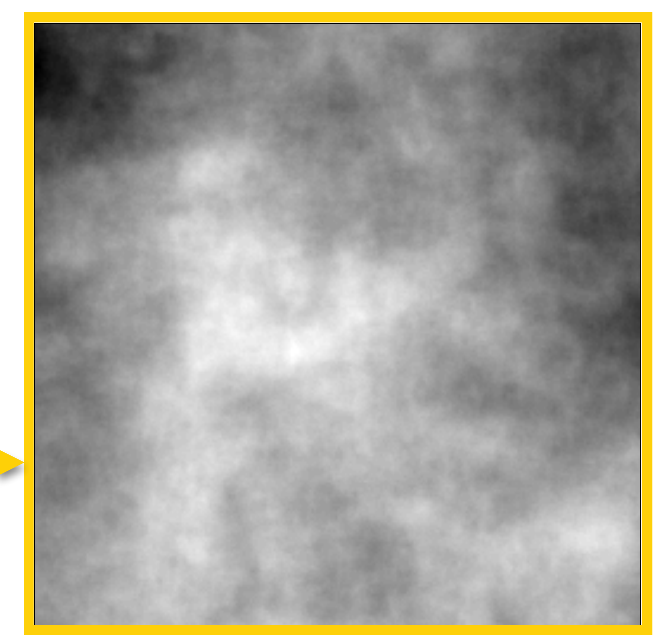


H=0.2

H=0.7



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anisotropic self-similar Gaussian fields

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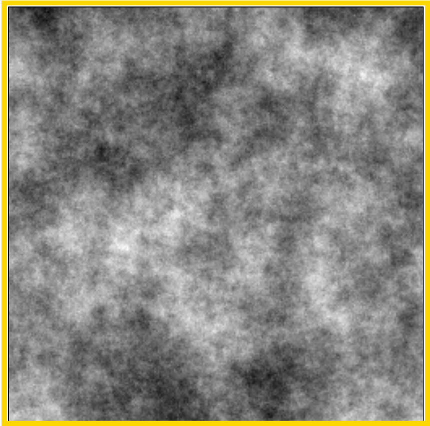

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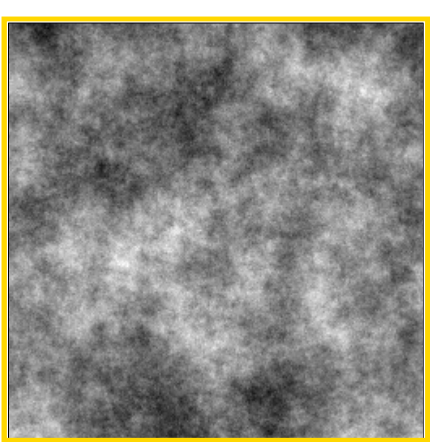

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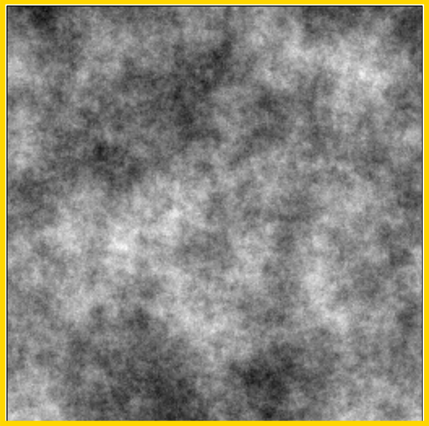

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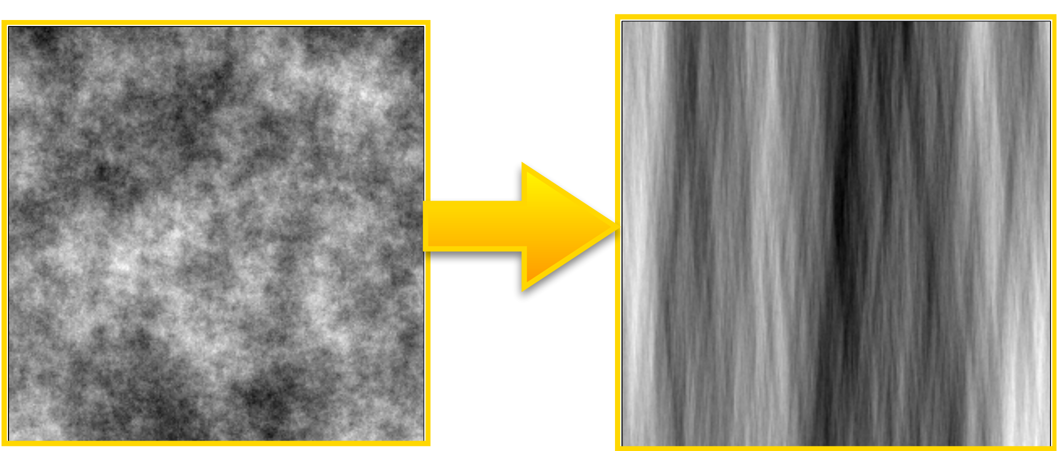

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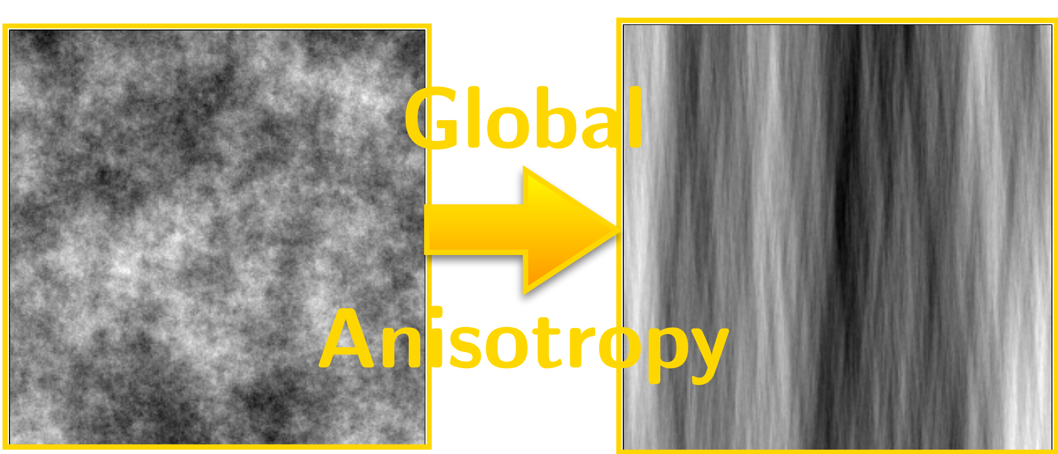

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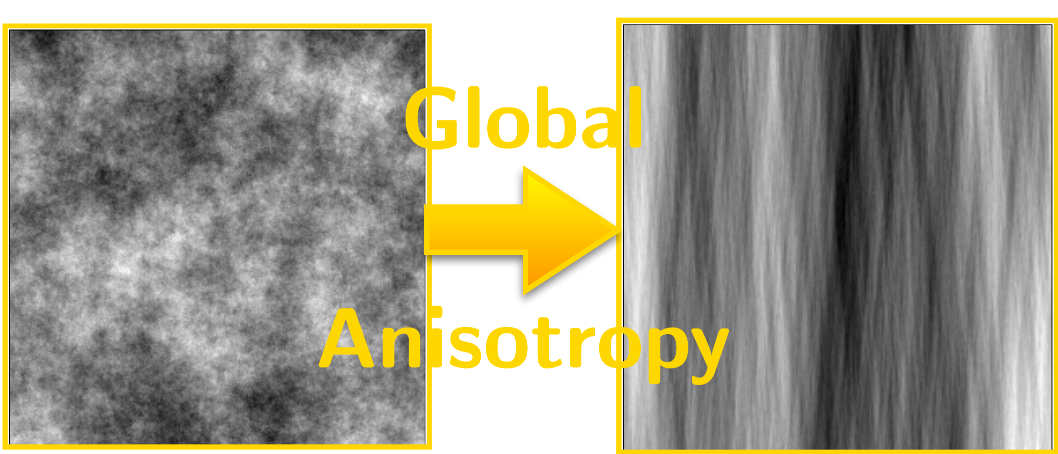

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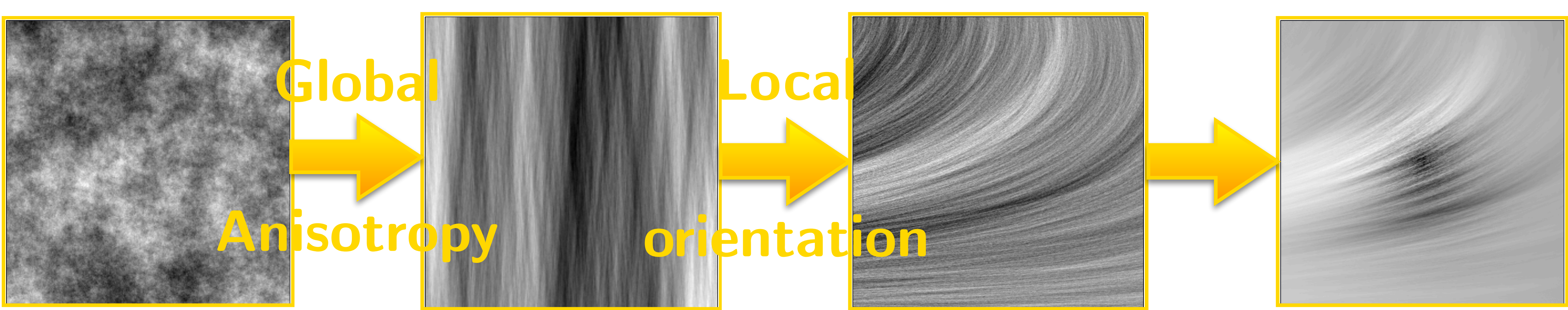

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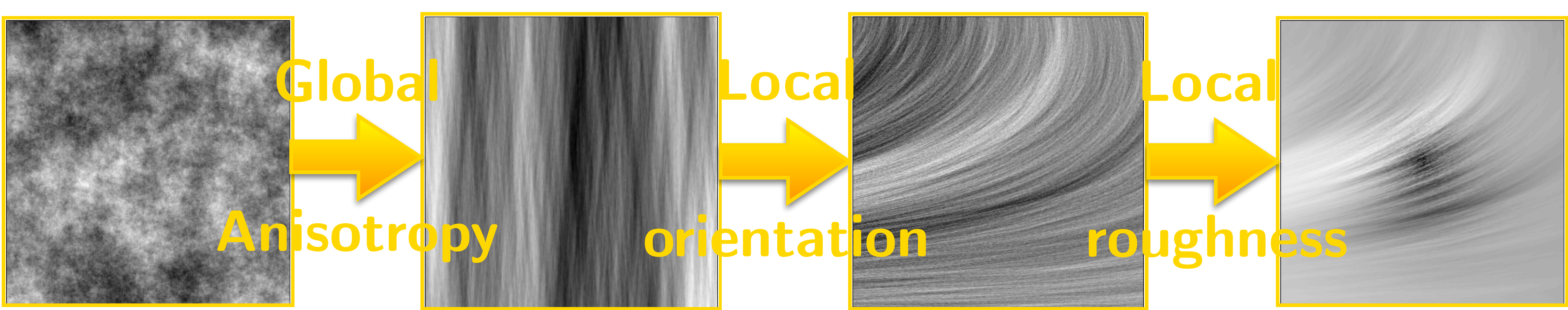

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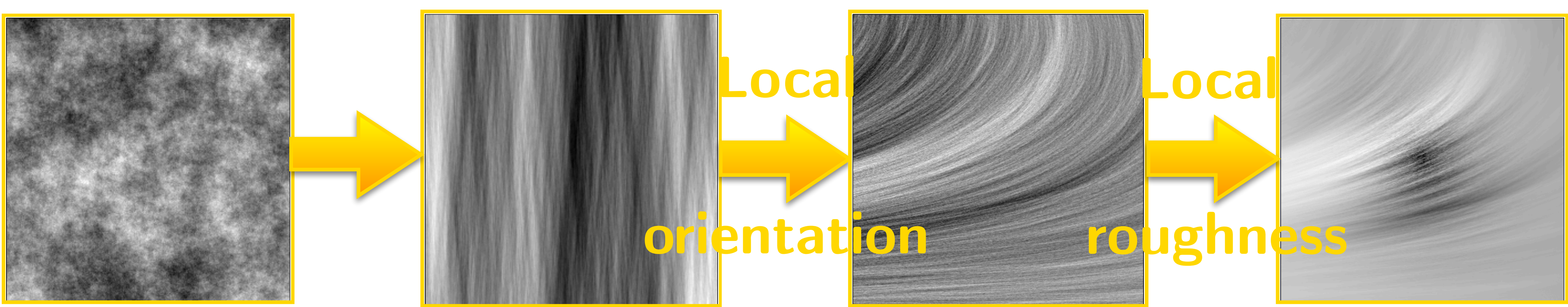

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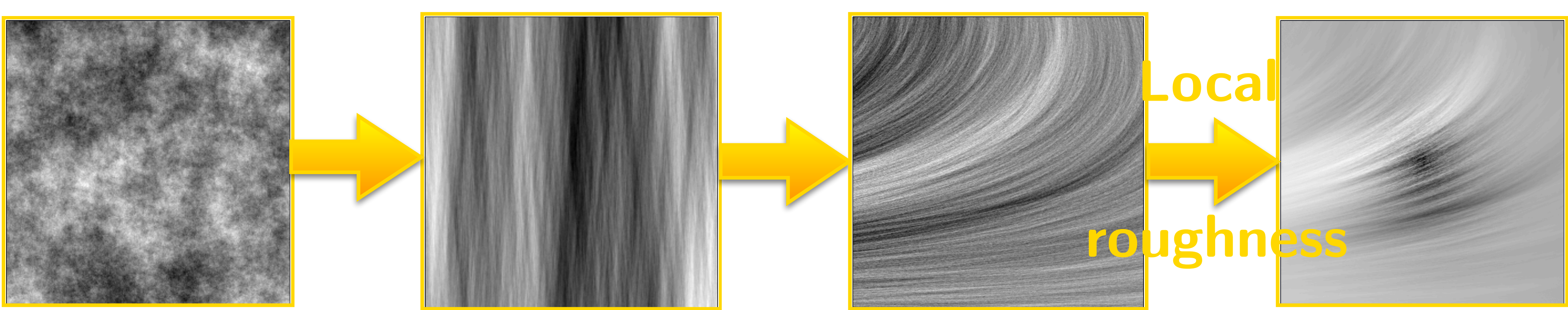

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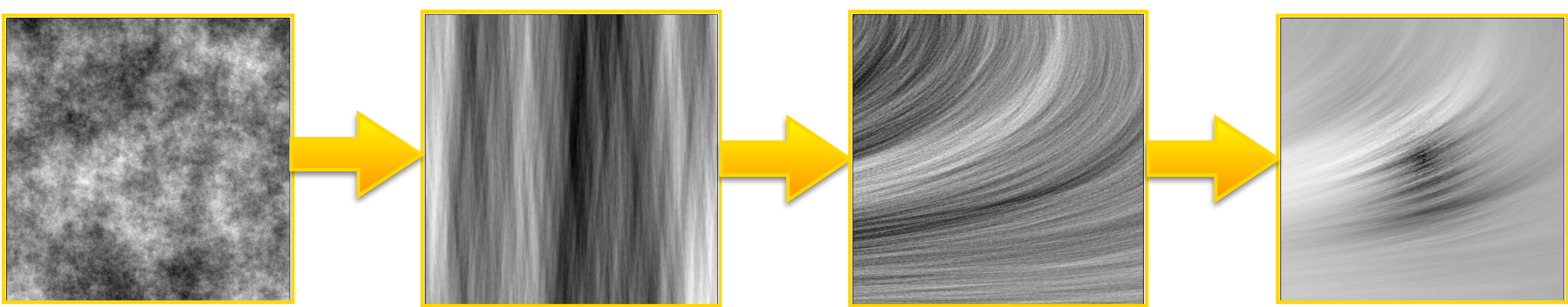

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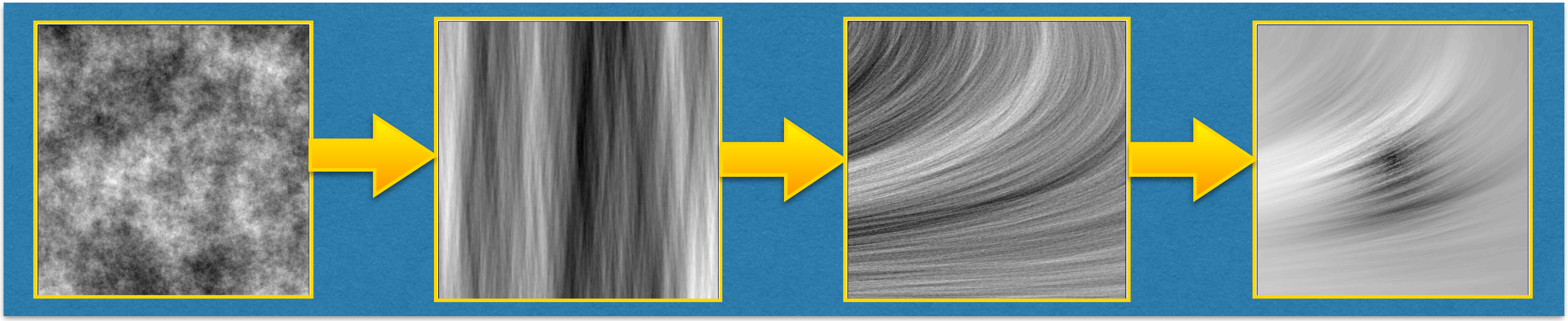
- $c(\mathbf{x},\xi) \equiv c(\mathbf{x},\arg\xi)$ and $h(\mathbf{x},\xi) \equiv h(\mathbf{x})$ [Polisano,Clausel,Perrier,Condat,2014]



Locally Anisotropic Fractional Brownian Field (LAFBF)

- **Definition**: Our new Gaussian model LAFBF is a local version of the elementary field

$$B^{H}_{\alpha_0,\alpha}(\mathbf{x}) = \int_{\mathbb{R}^2} (e^{i\mathbf{x}\cdot\xi} - 1) \frac{\mathbb{1}_{[-\alpha,\alpha]}(\arg\xi - \alpha_0(\mathbf{x}))}{\|\xi\|^{H+1}} d\widehat{W}(\xi)$$

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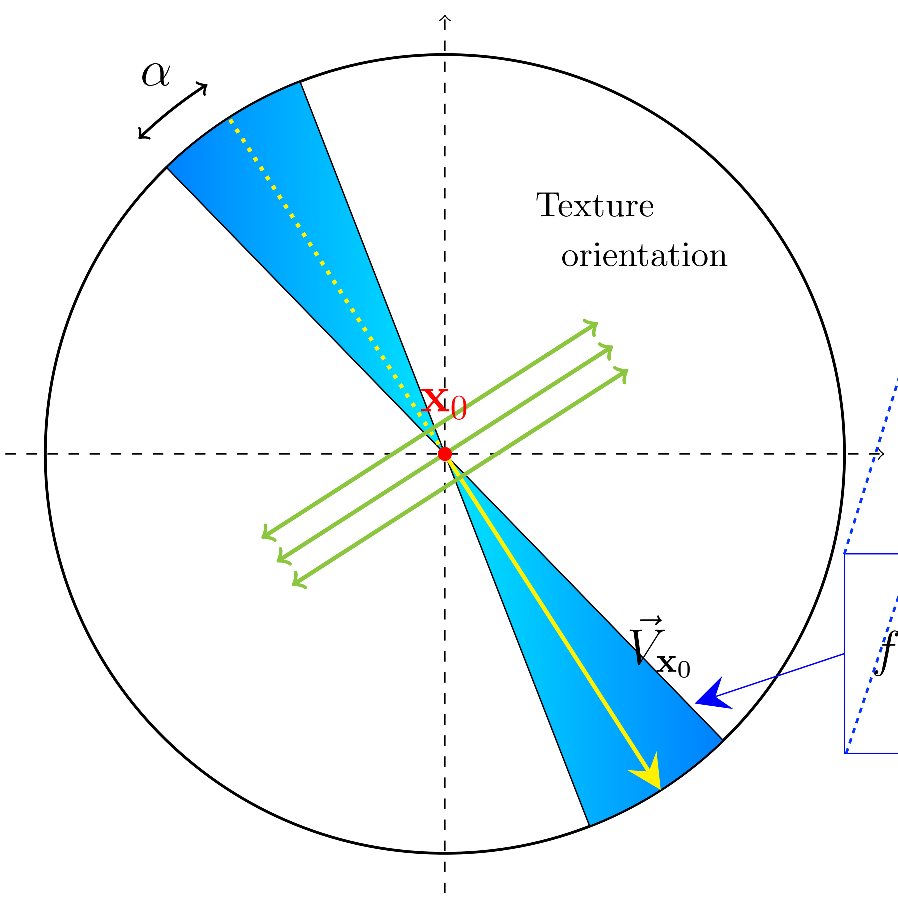
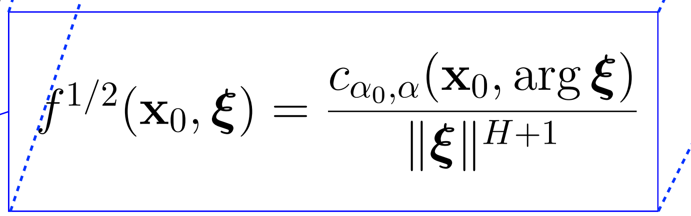
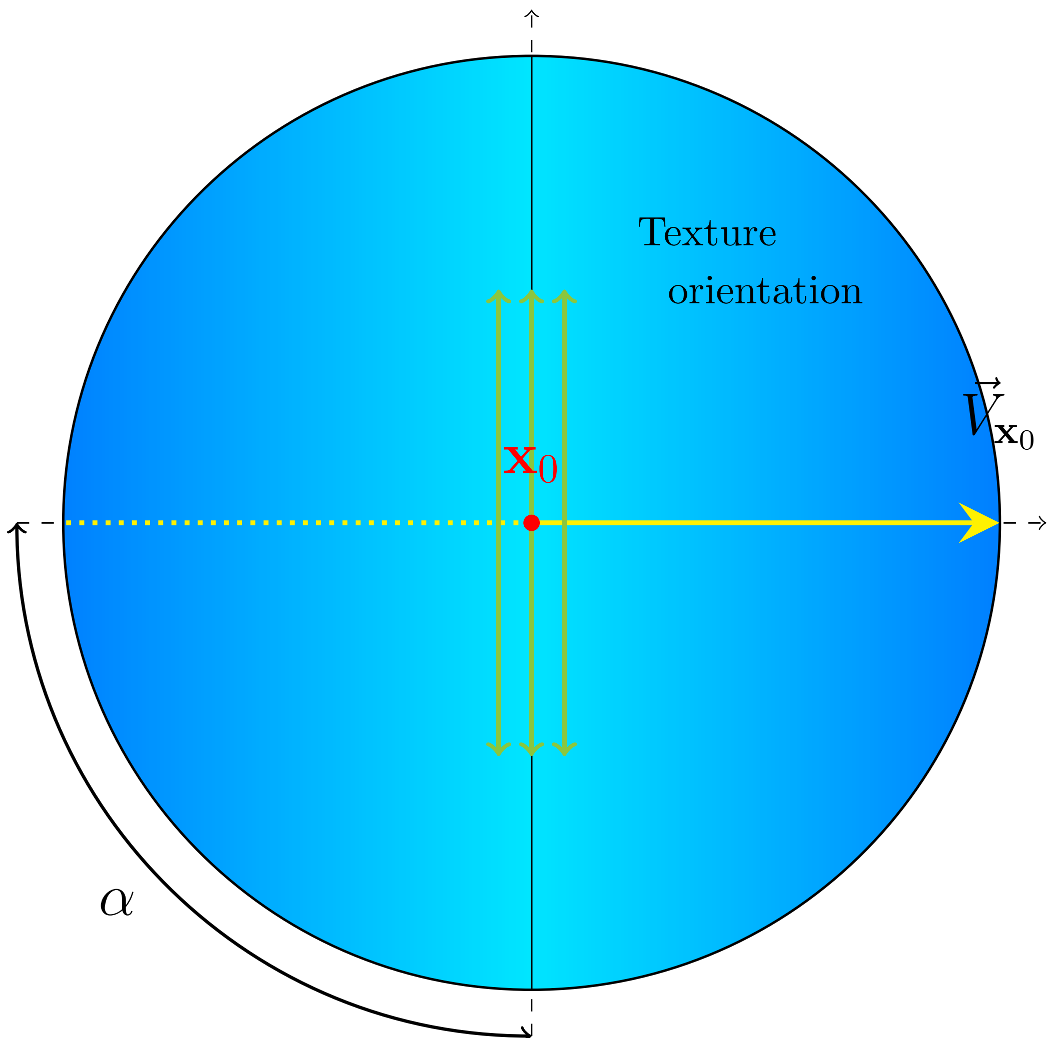


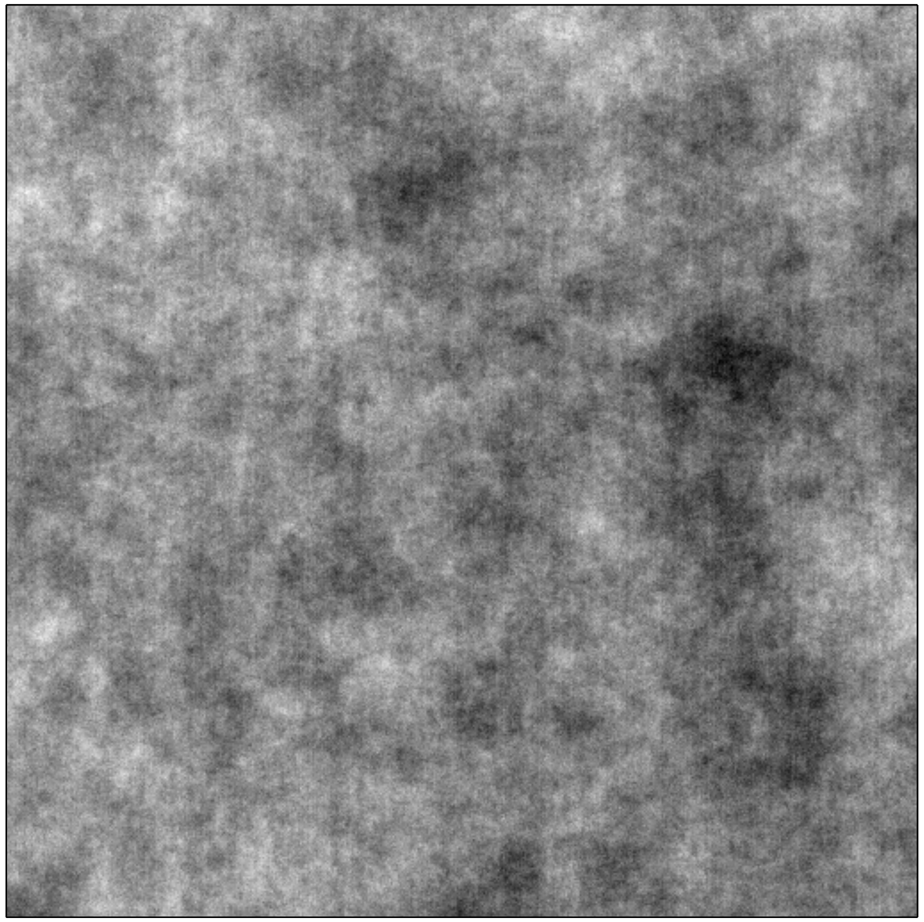


The orientation may varies spatially. α_0 is now a differentiable function on $\mathbb{R}^2$

Elementary field

$$\alpha_0 = 0$$





$$\alpha = -\frac{\pi}{2}$$

Elementary field

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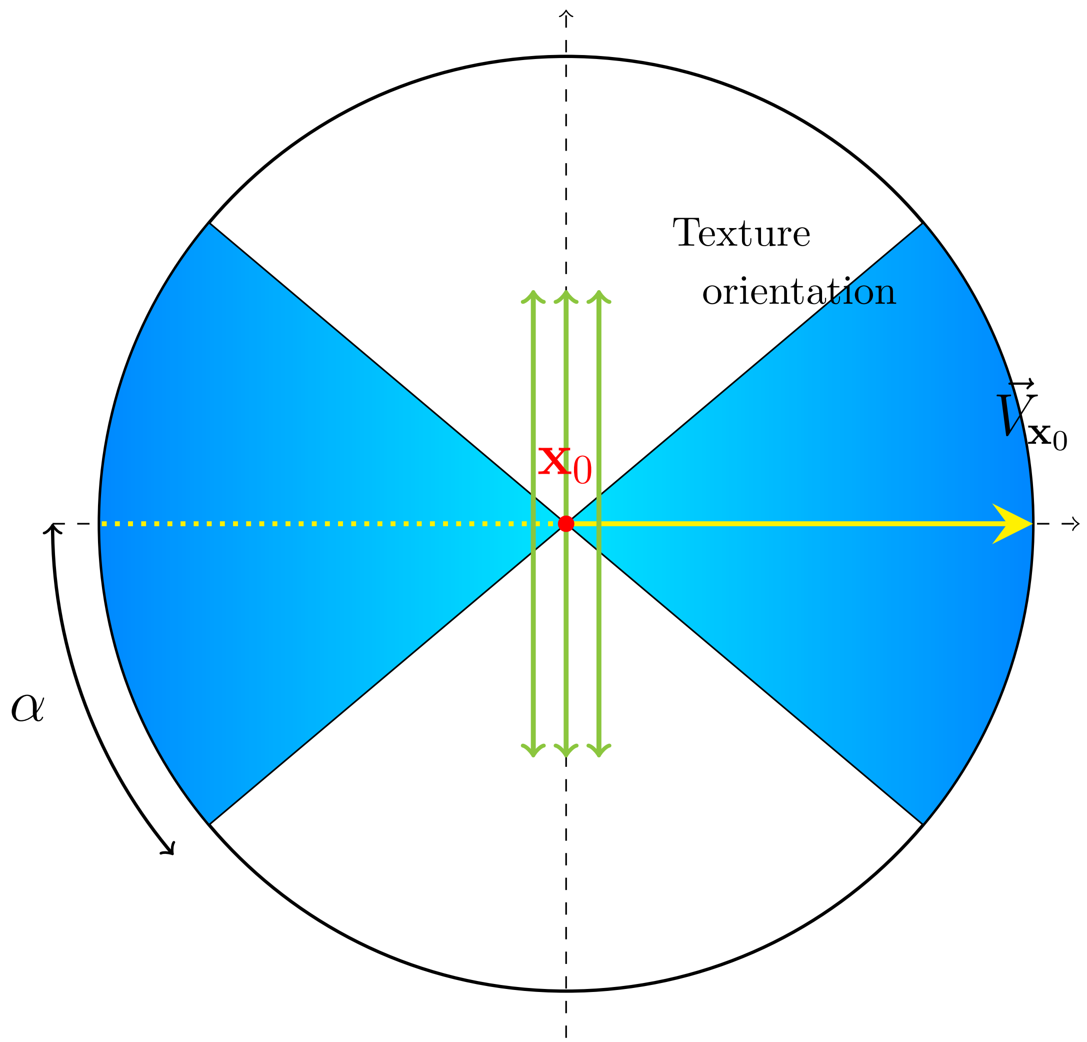
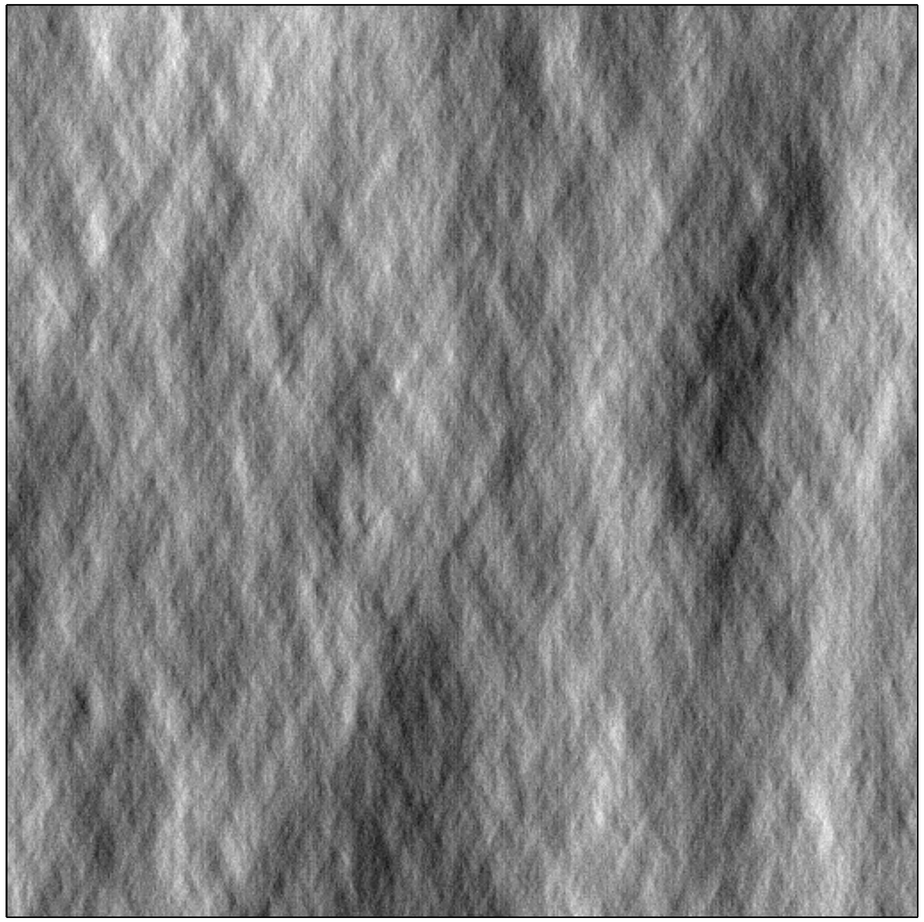
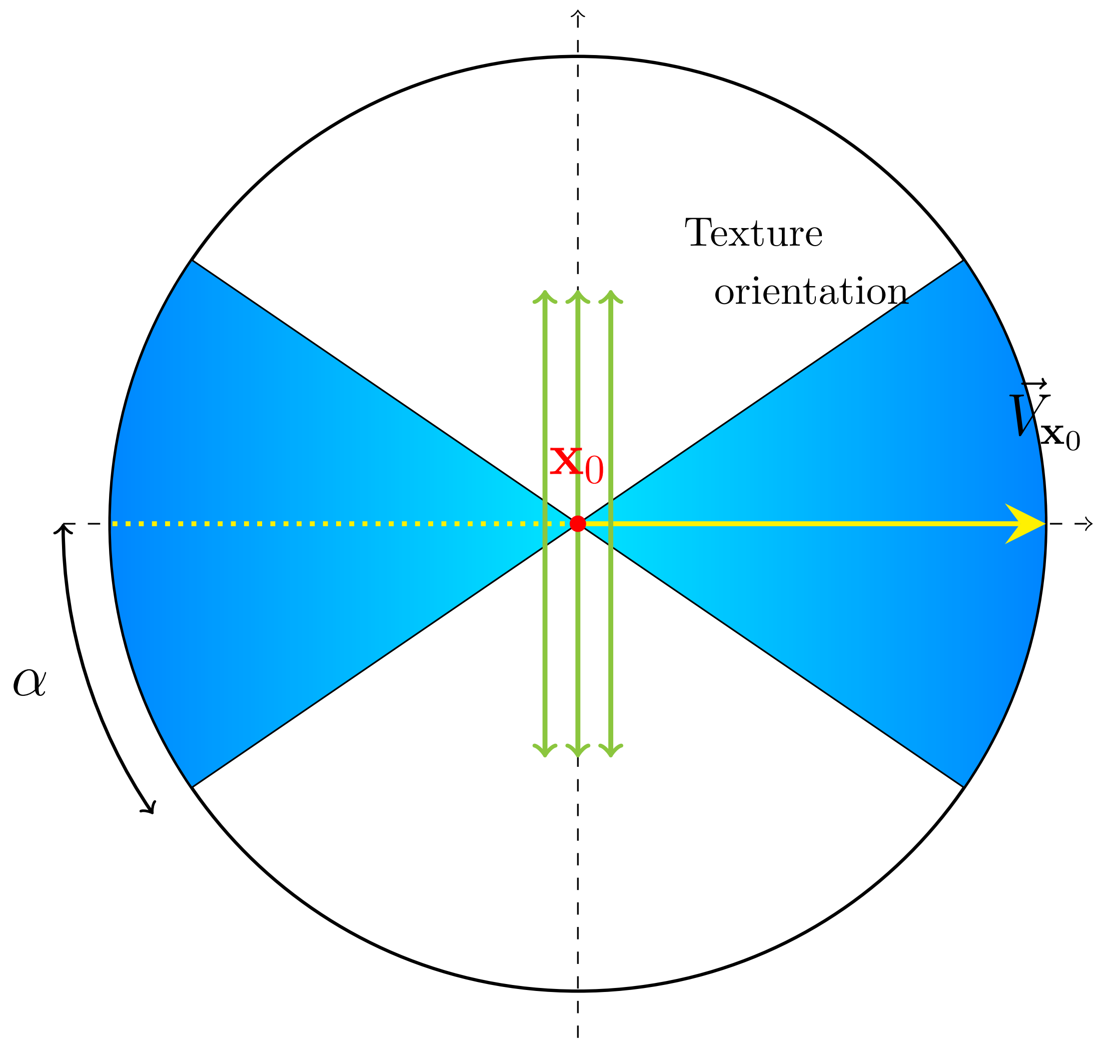


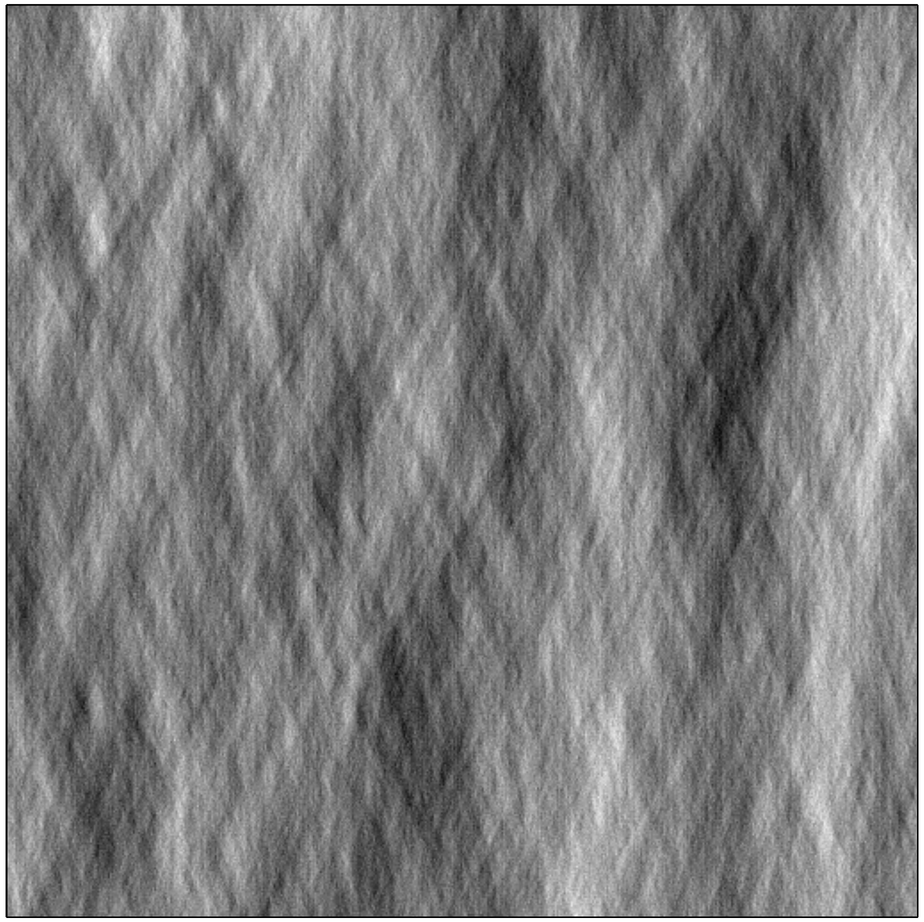


$\alpha = 0.7$

Elementary field

$\alpha_0 = 0$

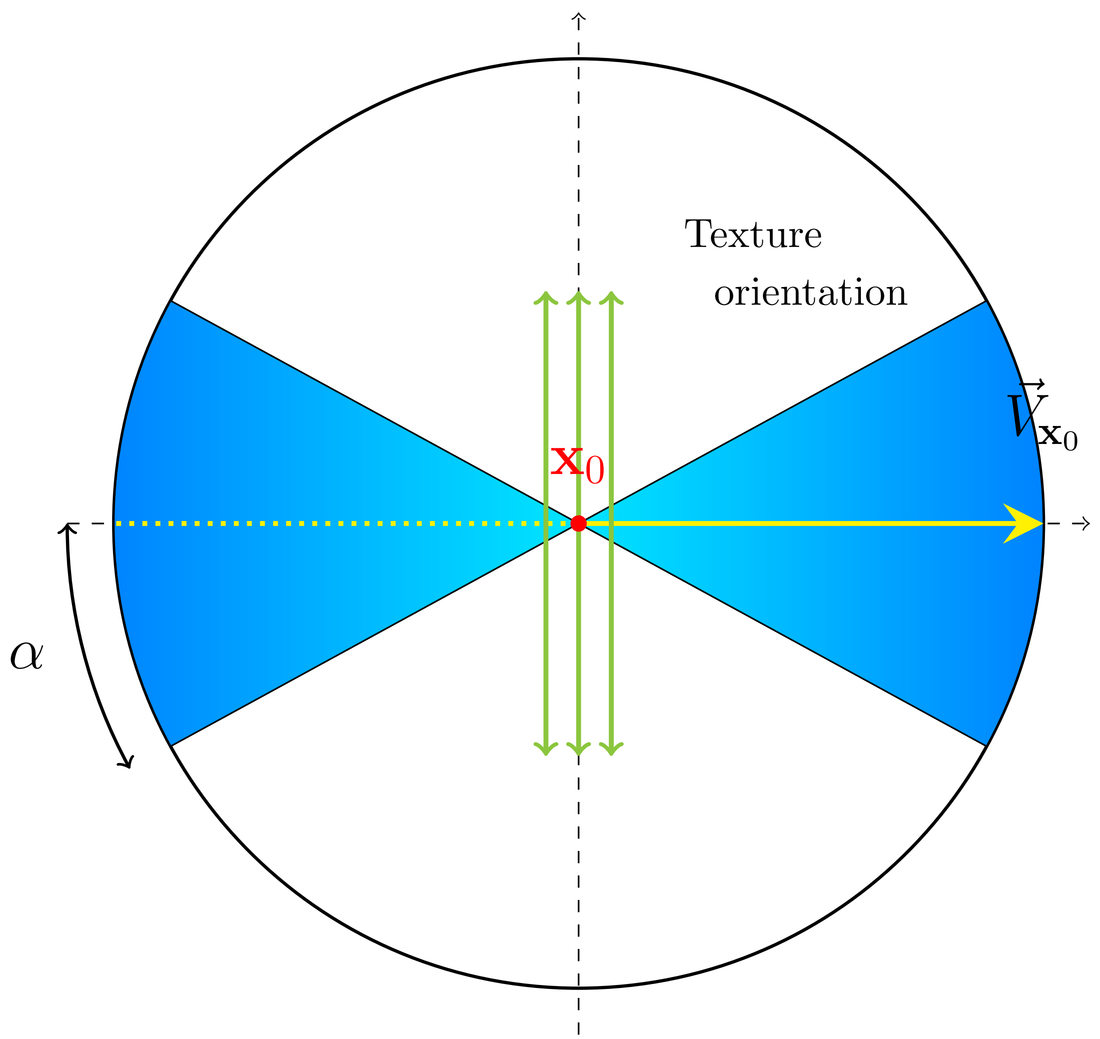


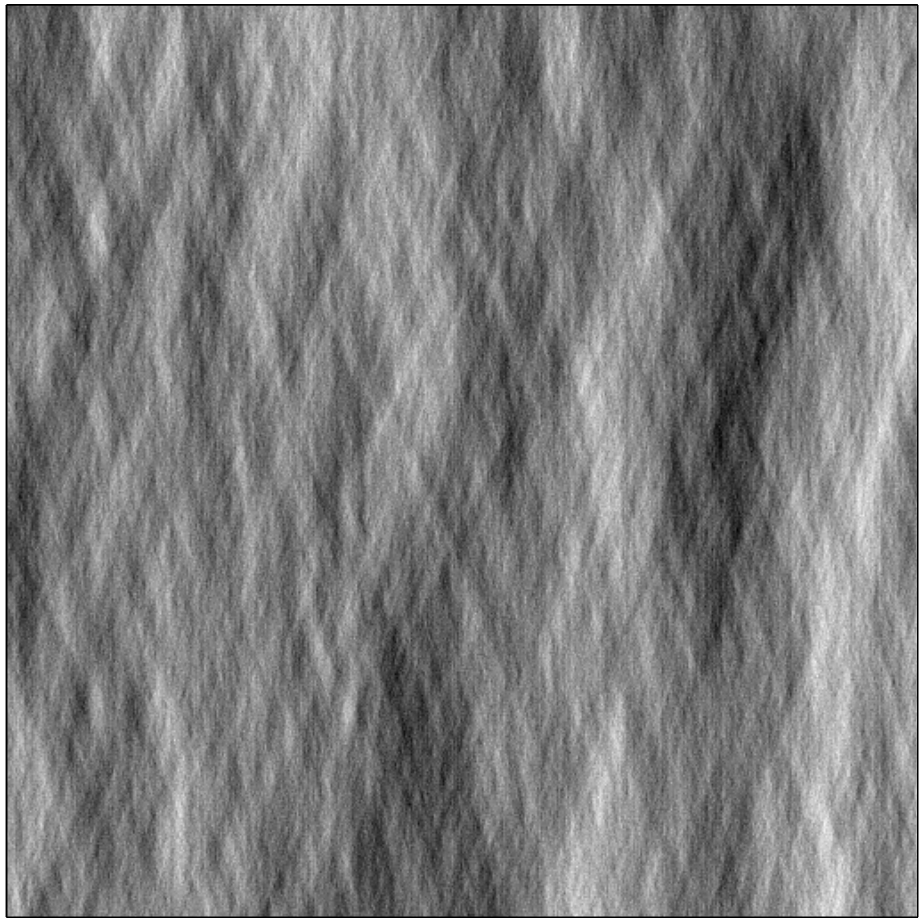


$\alpha = 0.6$

Elementary field

$\alpha_0 = 0$

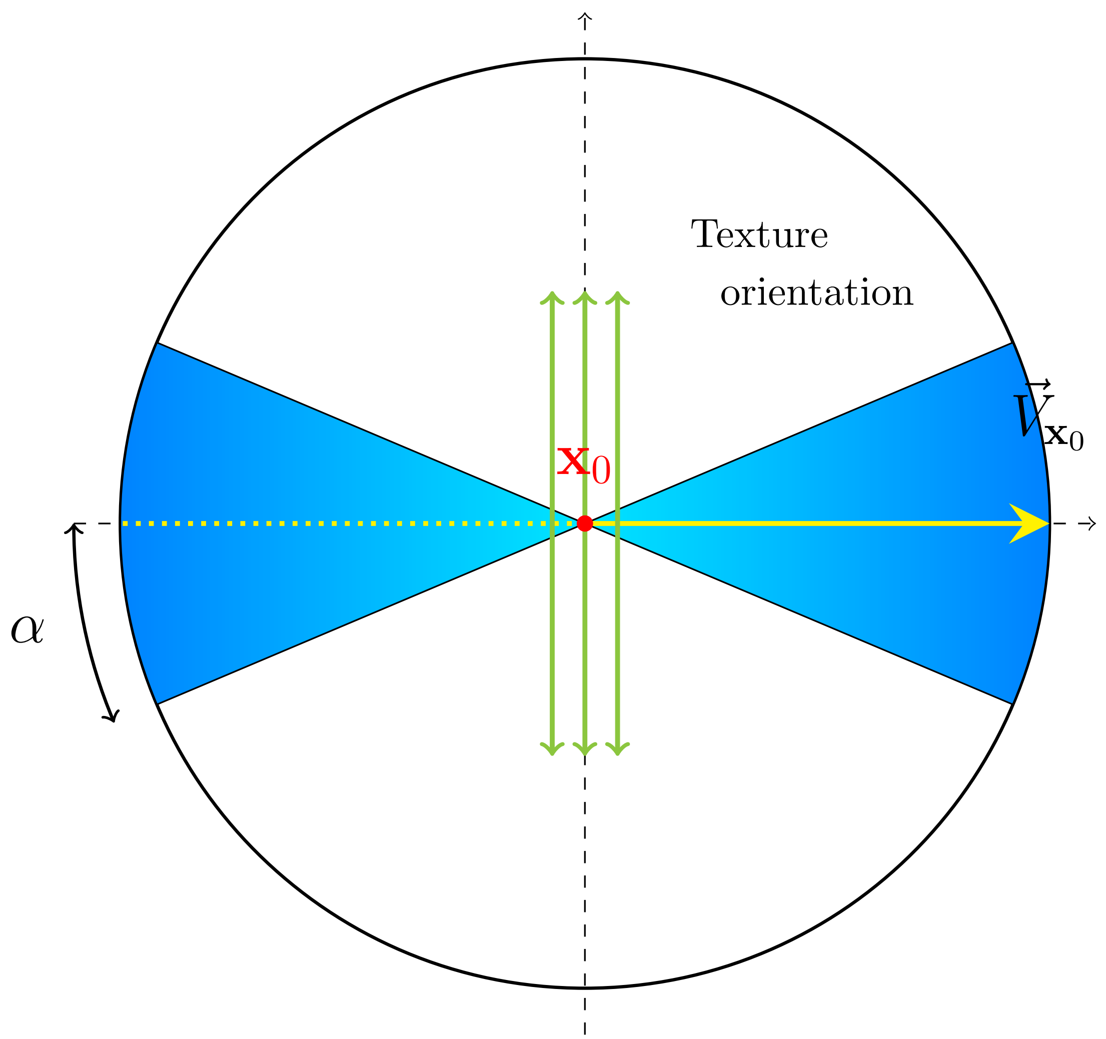


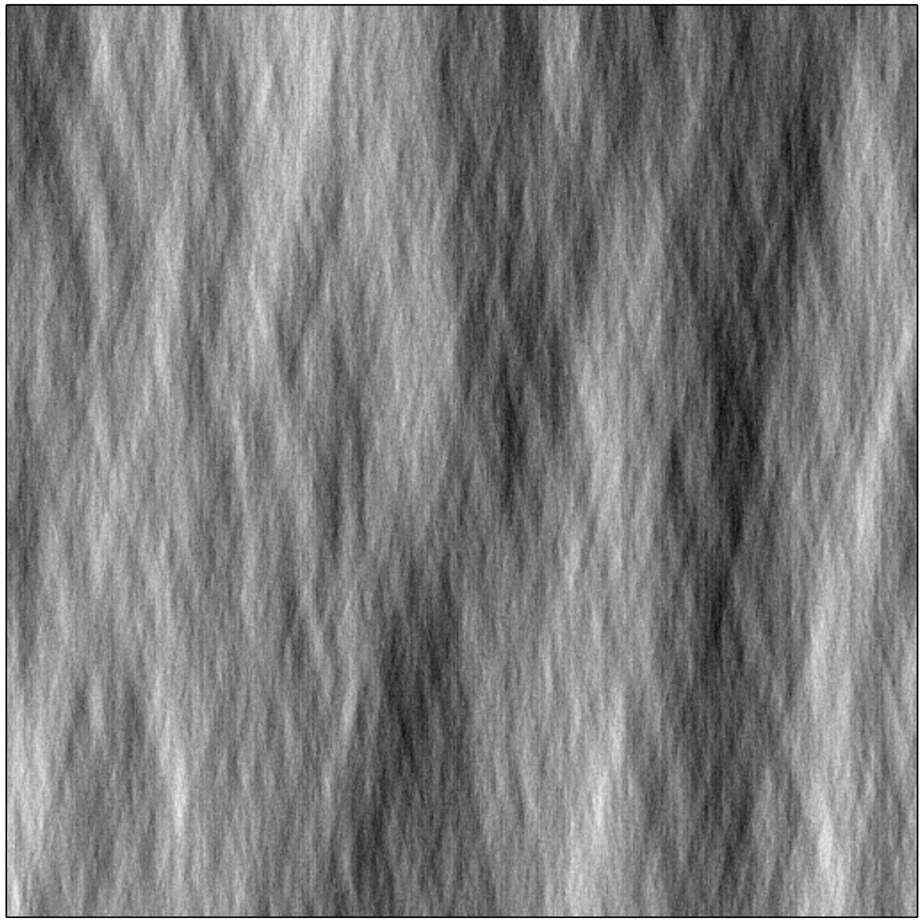


$\alpha = 0.5$

Elementary field

$\alpha_0 = 0$

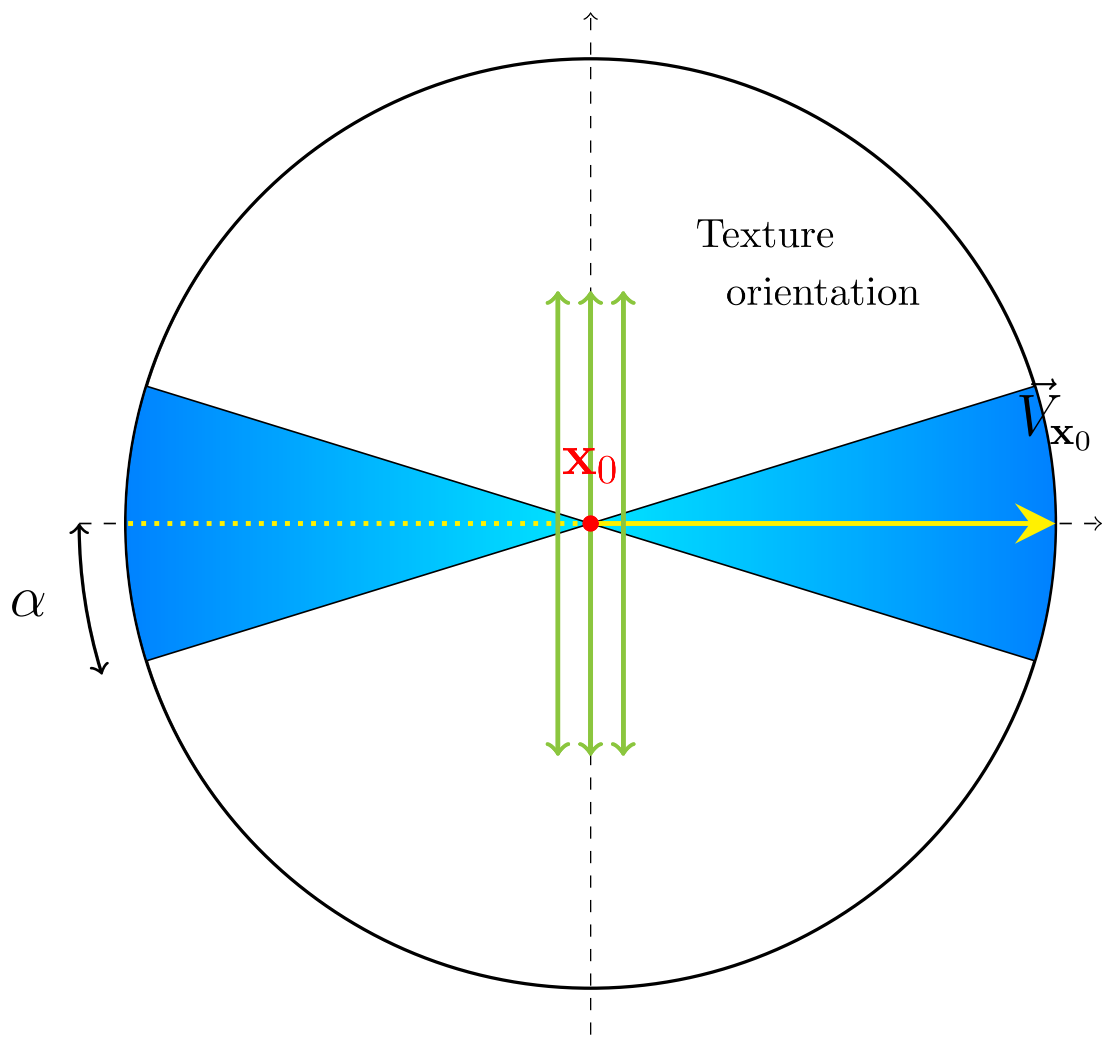


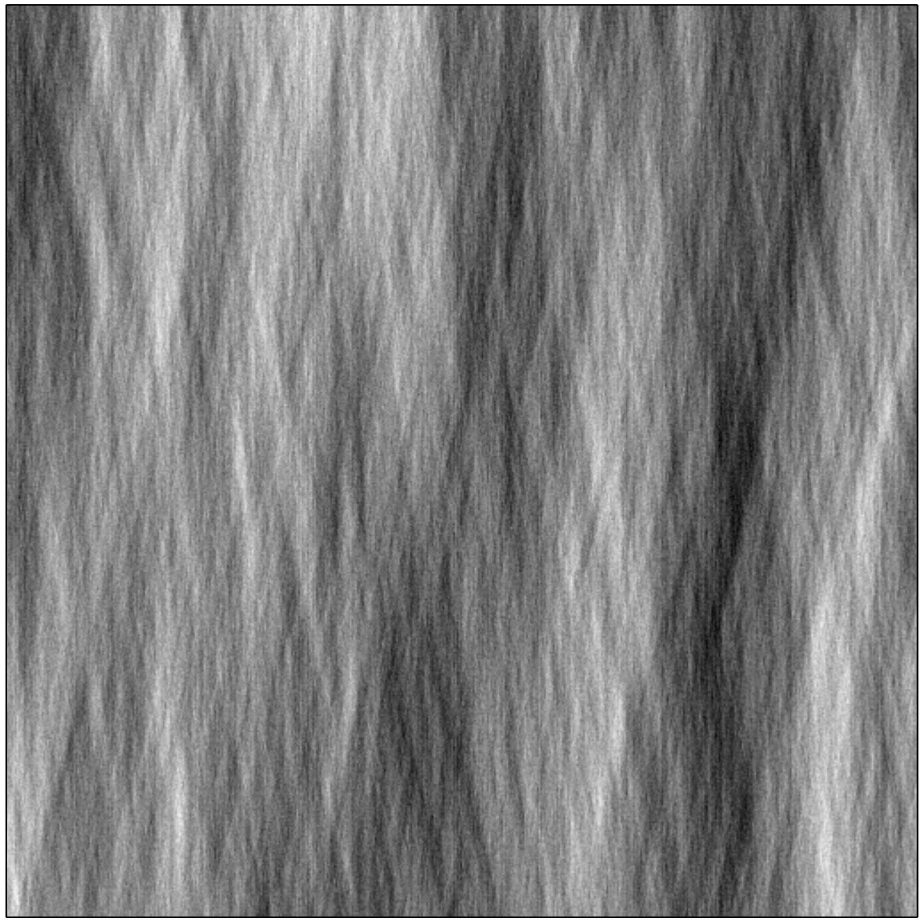


$\alpha = 0.4$

Elementary field

$\alpha_0 = 0$

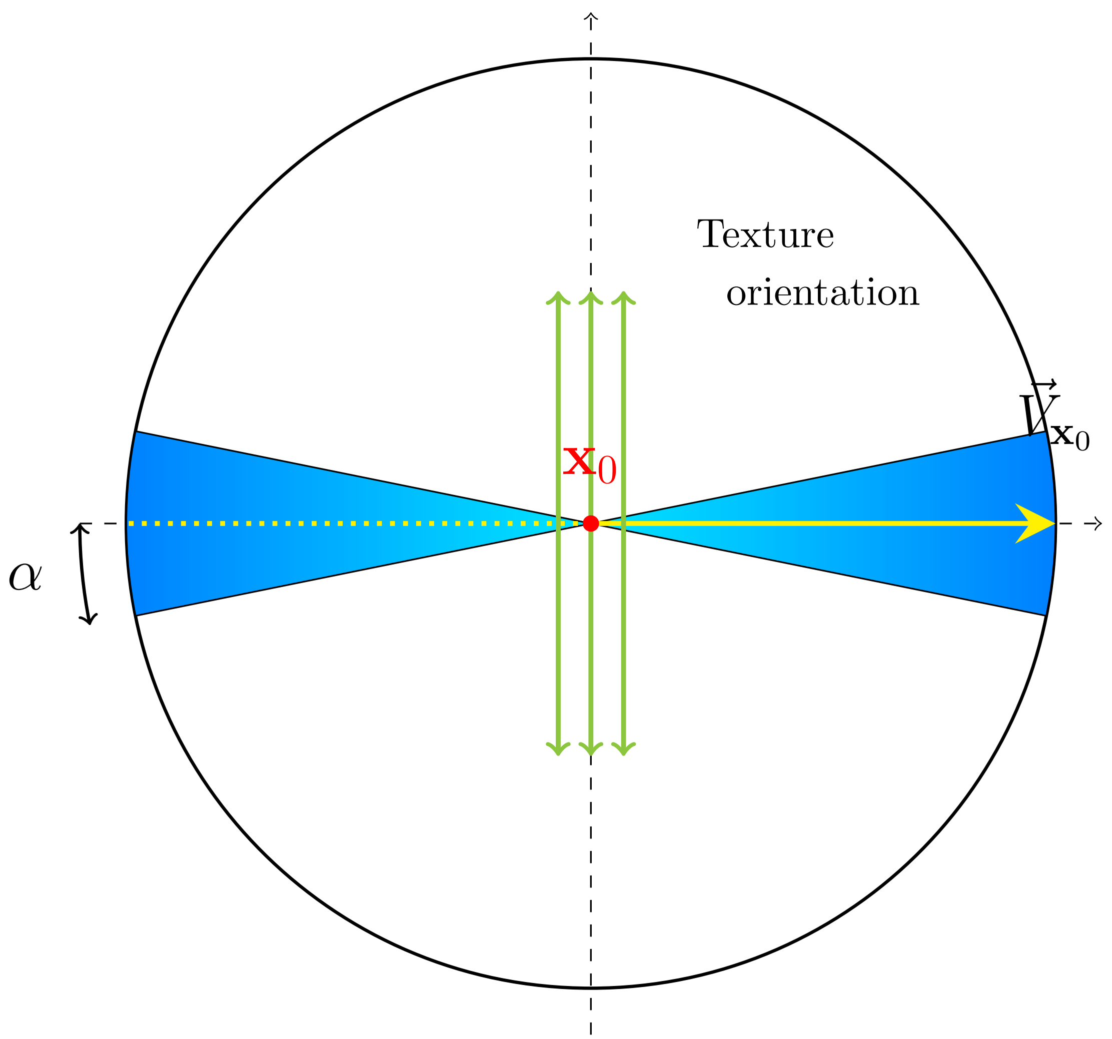




$\alpha = 0.3$

Elementary field

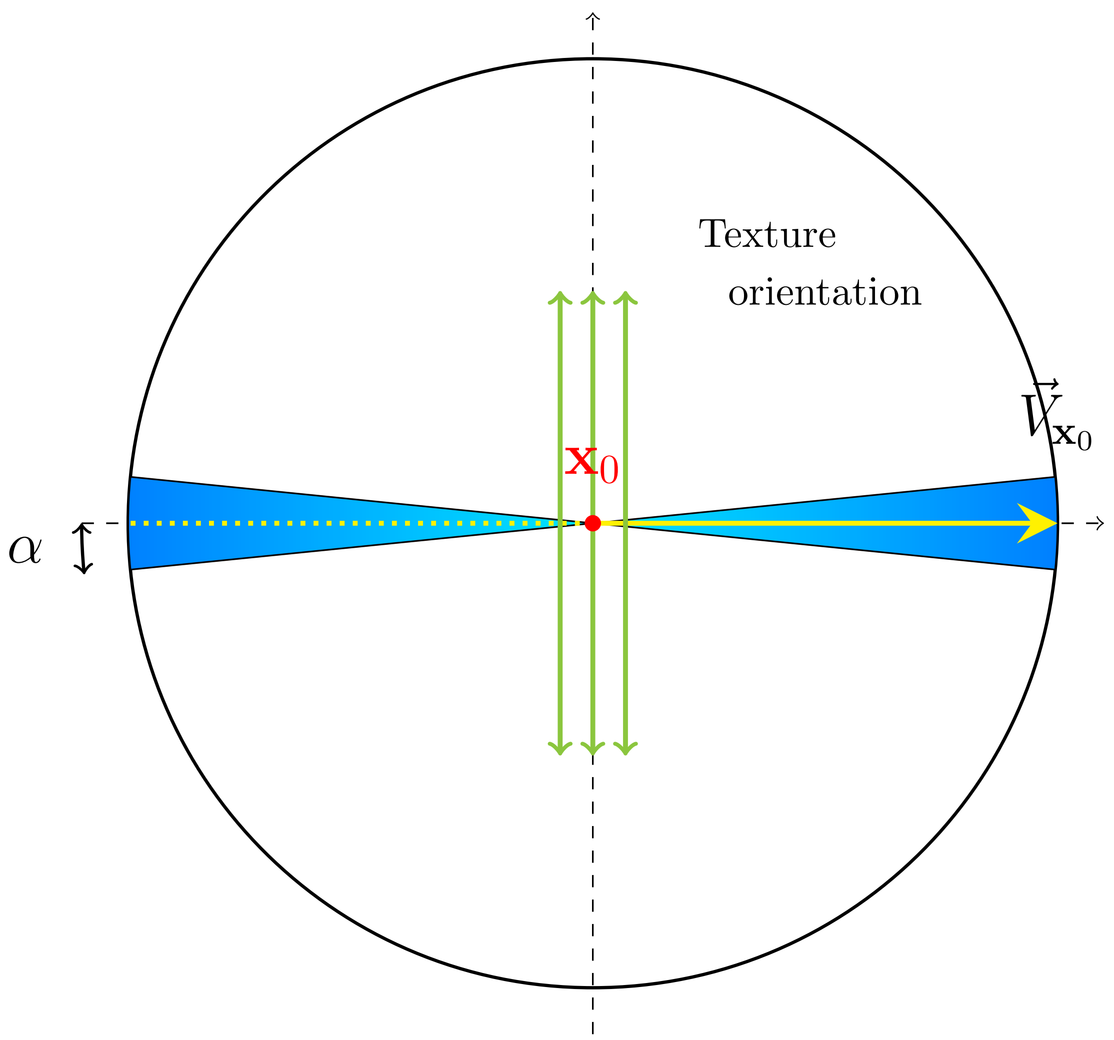
$\alpha_0 = 0$

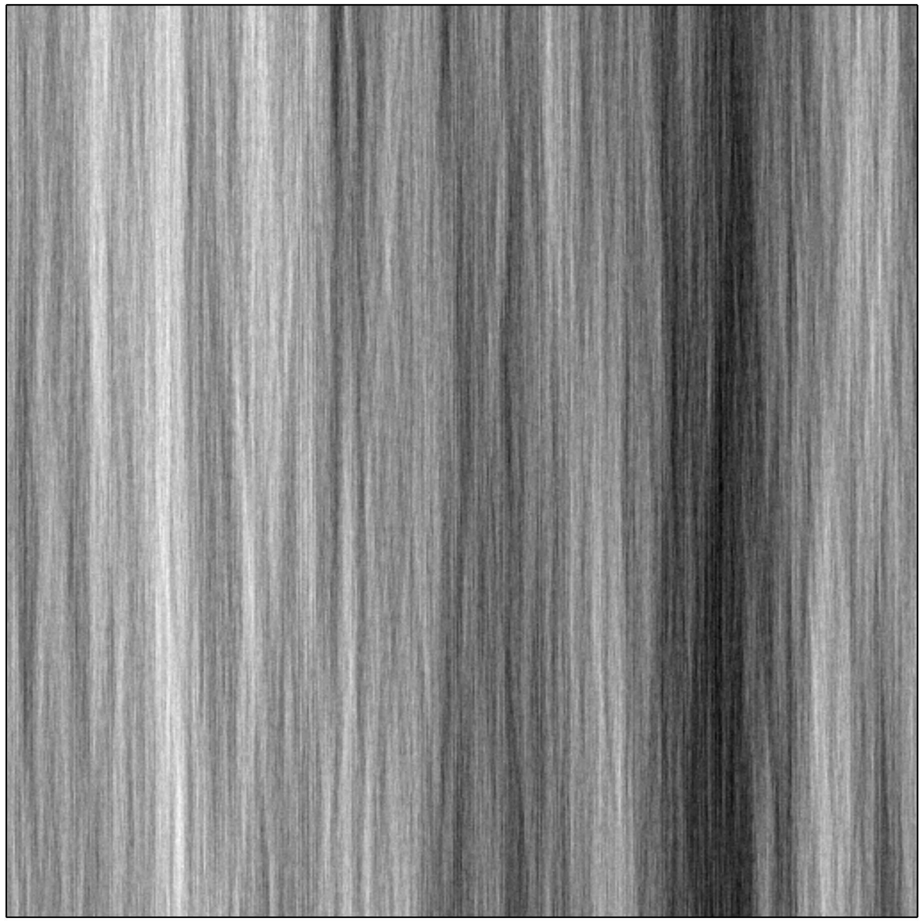


$\alpha = 0.2$

Elementary field

$\alpha_0 = 0$

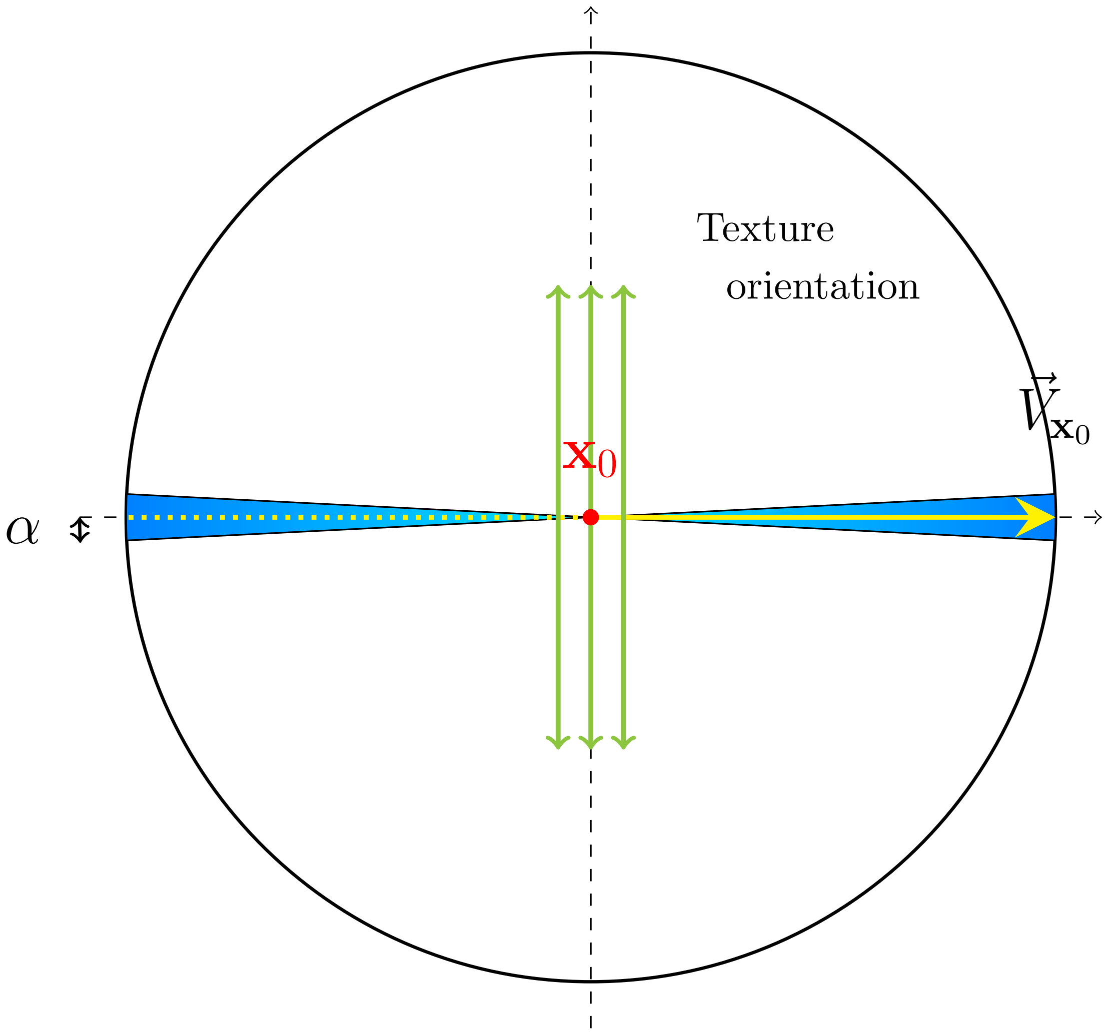


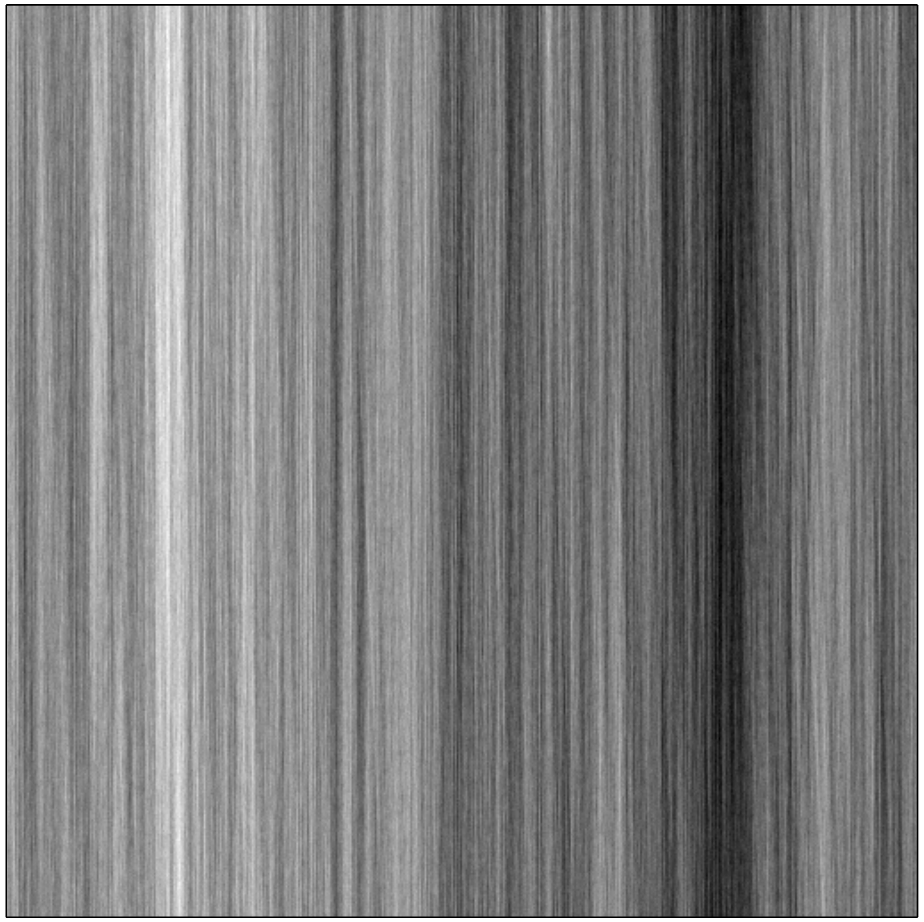


$\alpha = 0.1$

Elementary field

$\alpha_0 = 0$

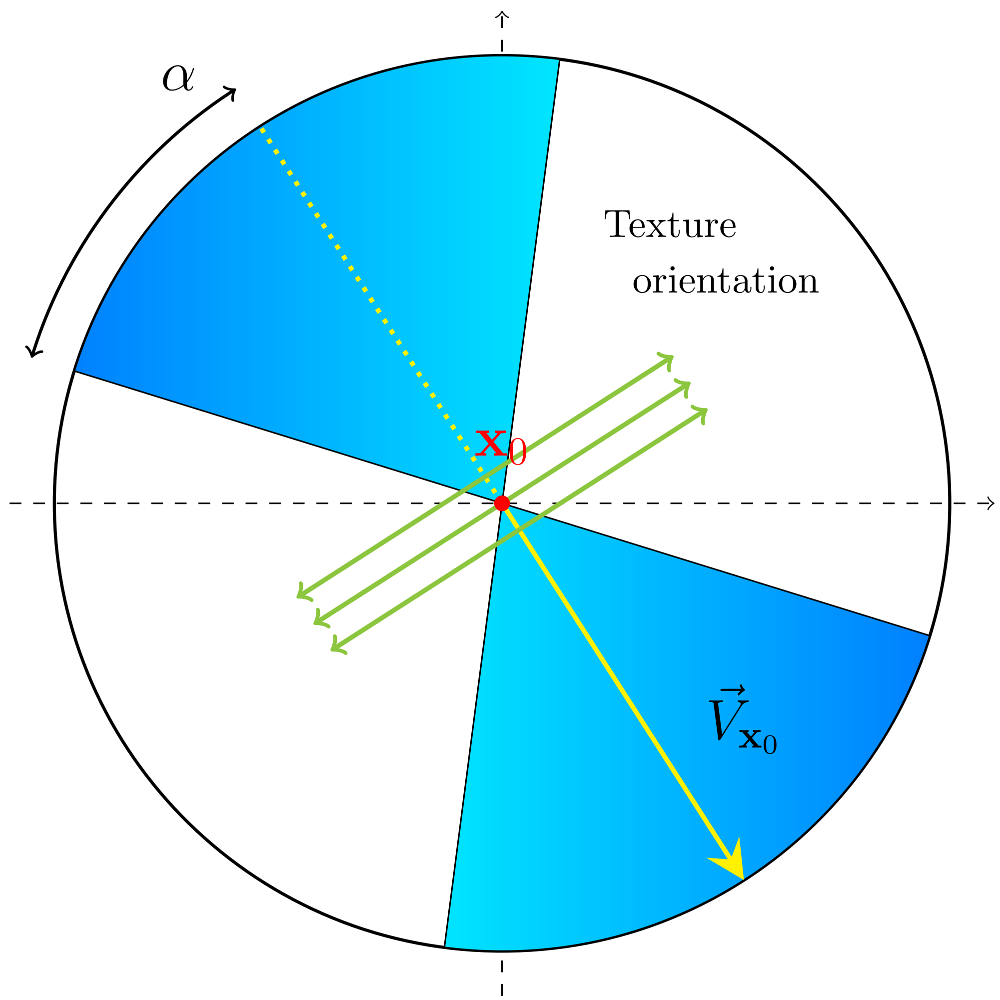


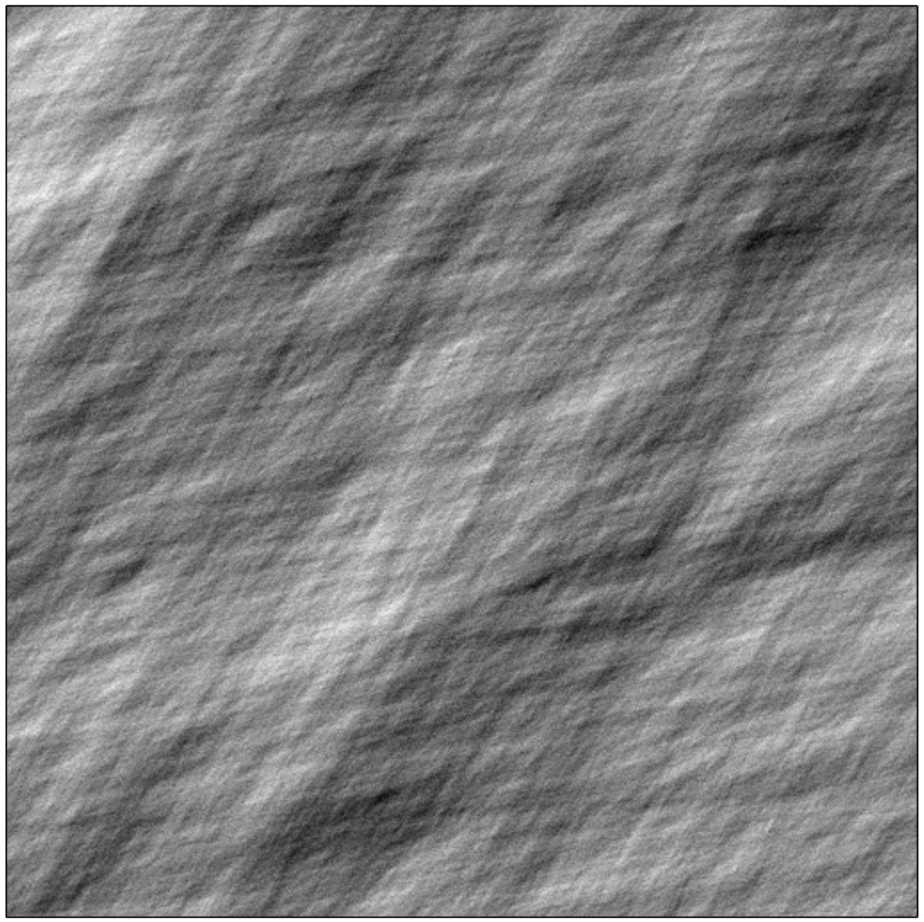


$\alpha = 0.05$

Elementary field

$$\alpha_0 = -\frac{\pi}{3}$$





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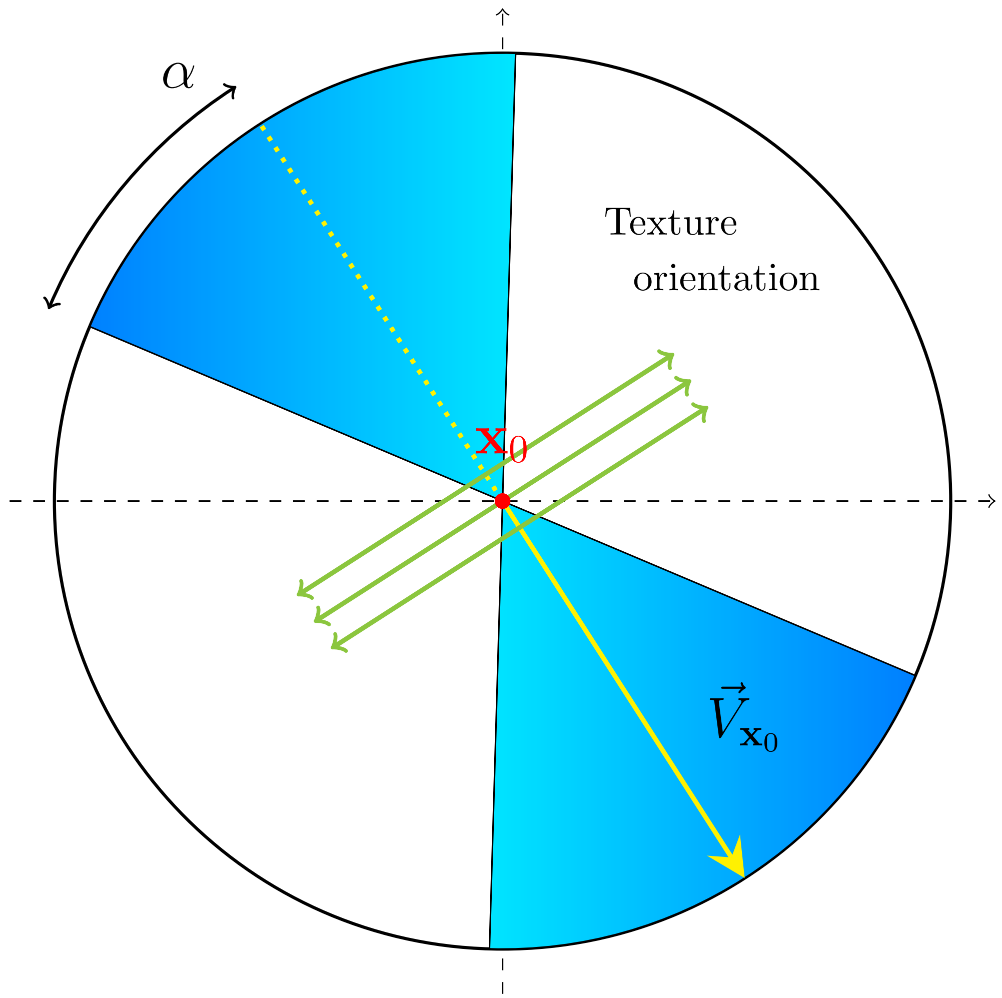

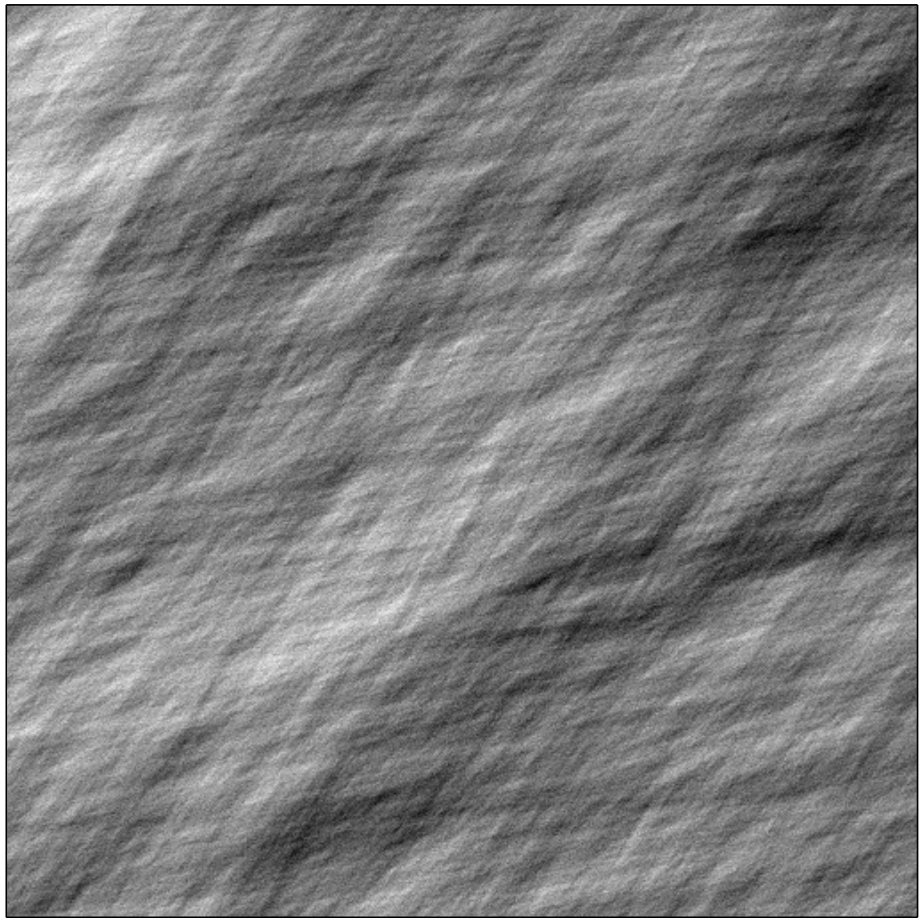

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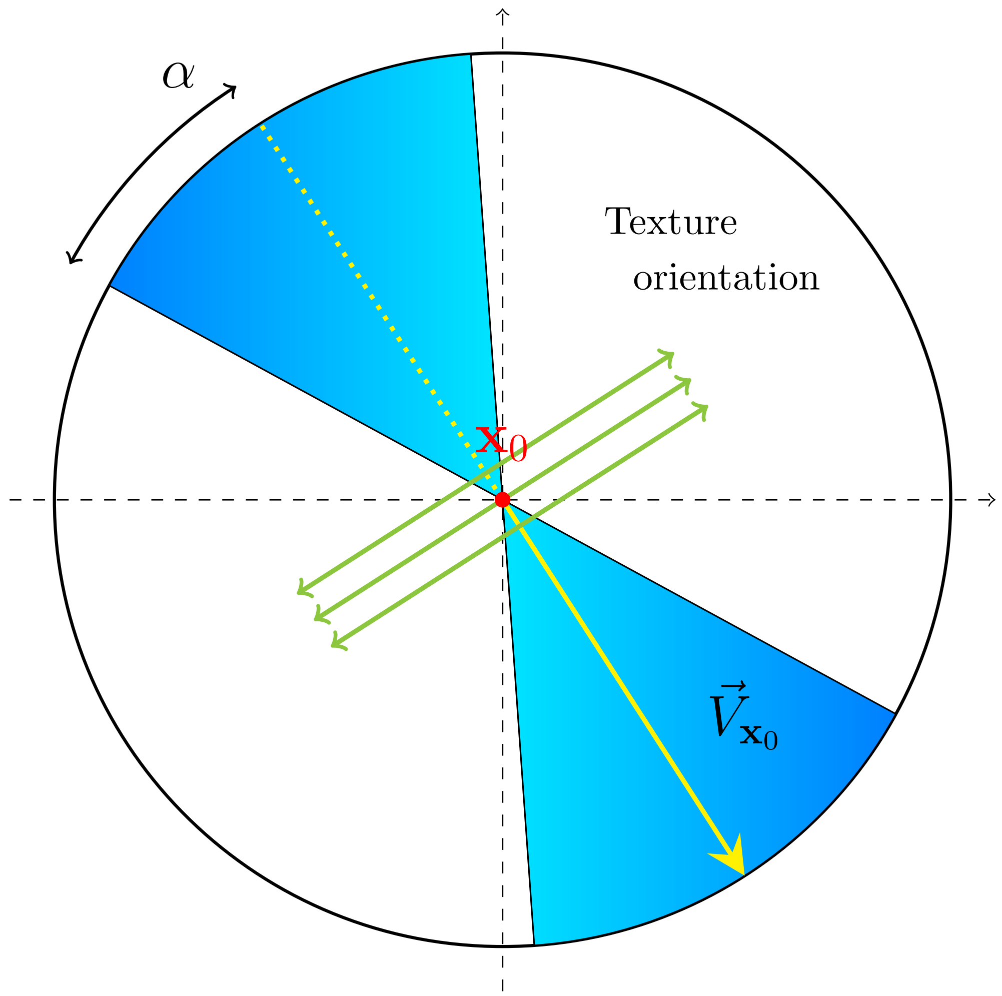

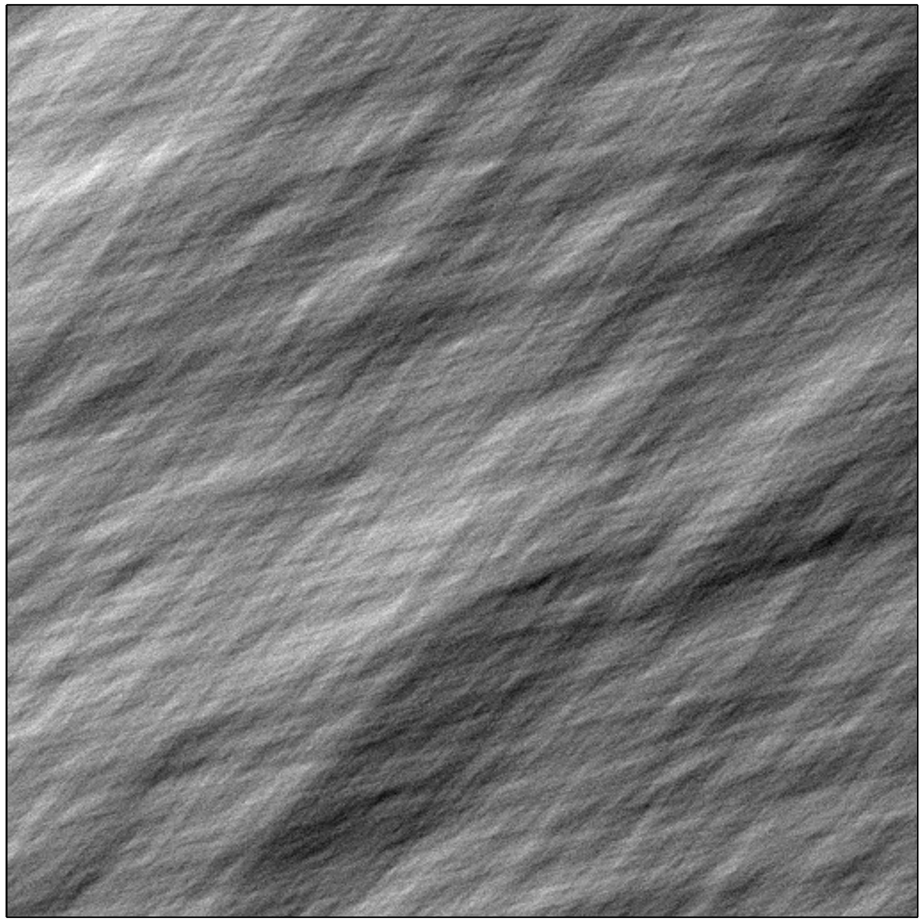

$$\alpha = 0.5$$

Elementary field

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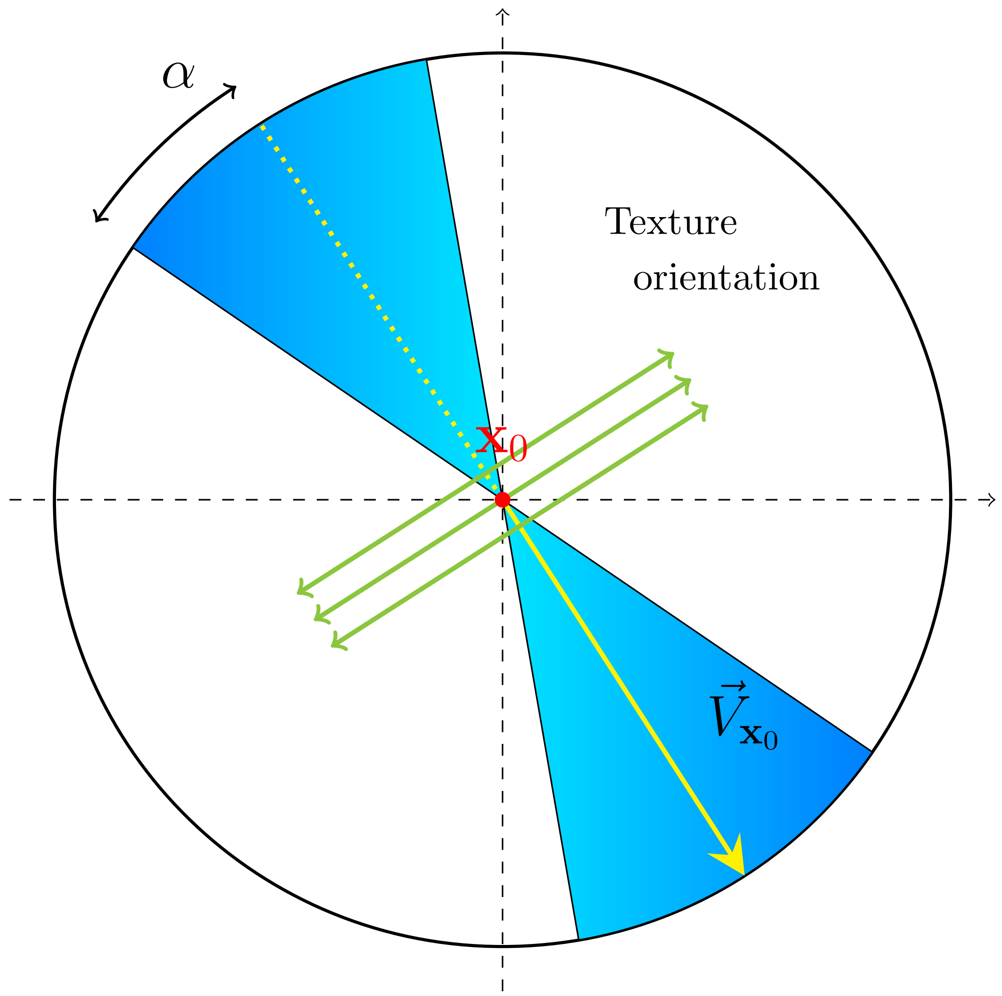

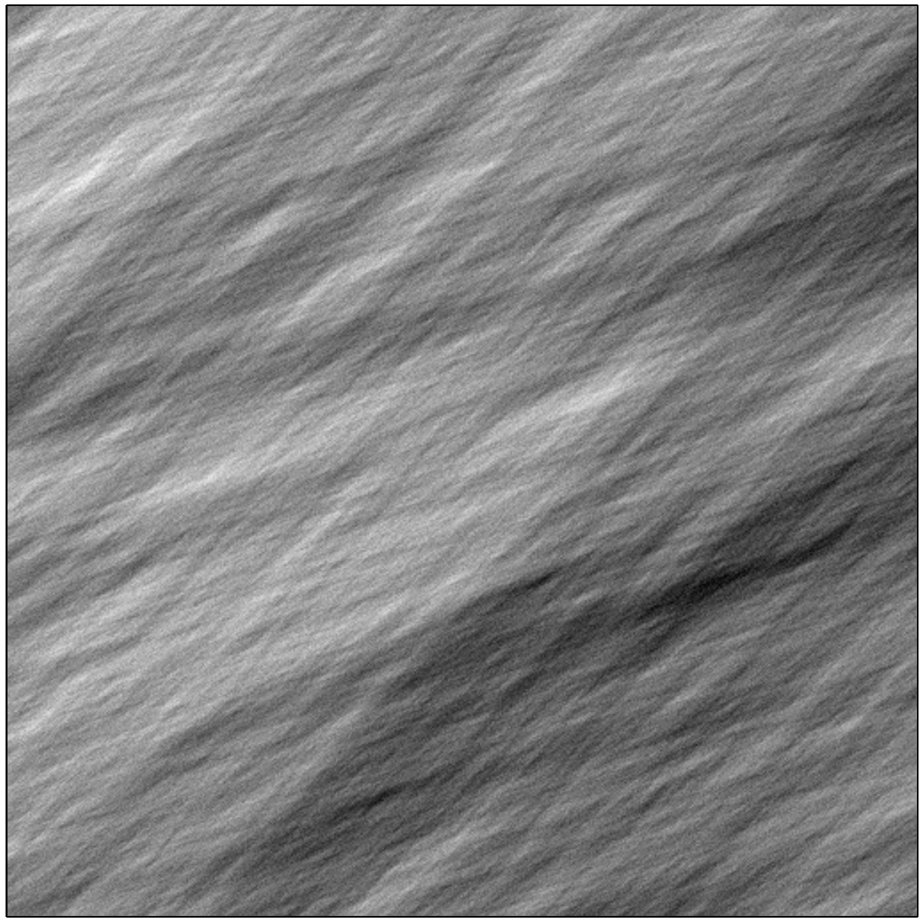

$$\alpha = 0.4$$

Elementary field

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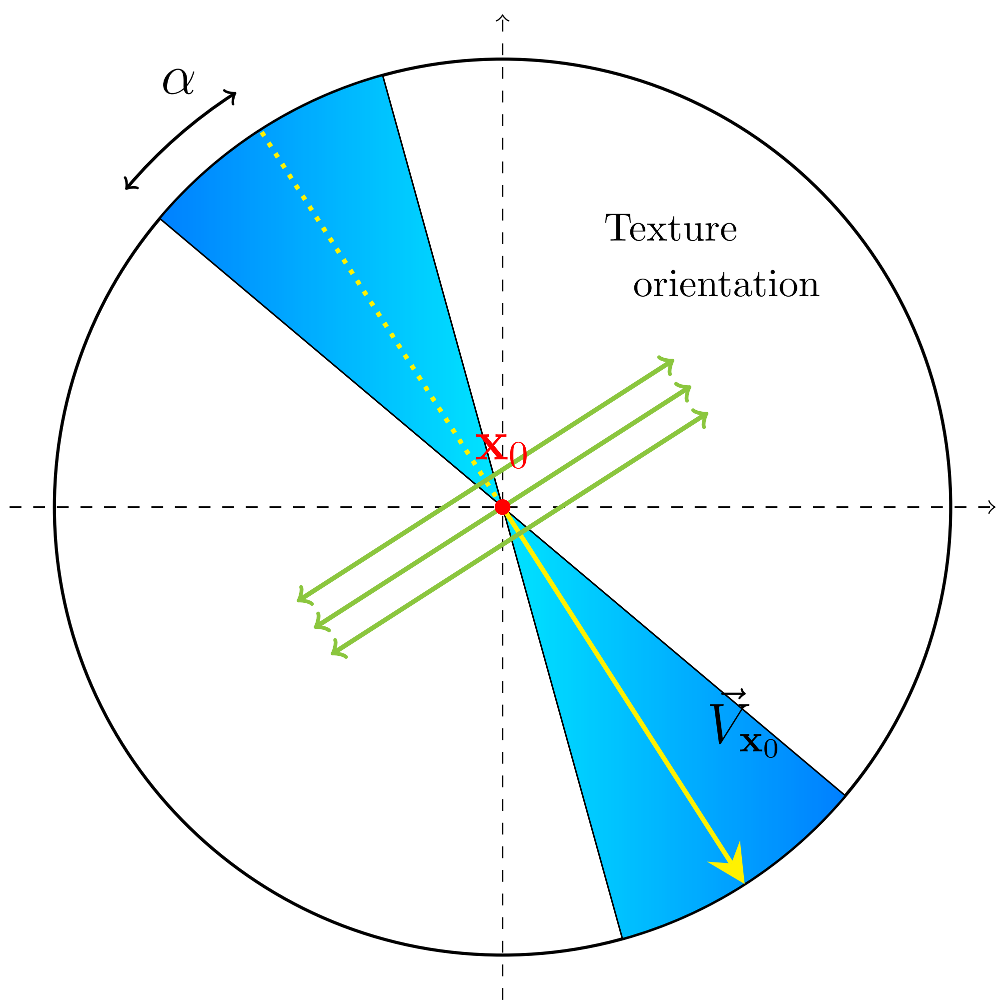

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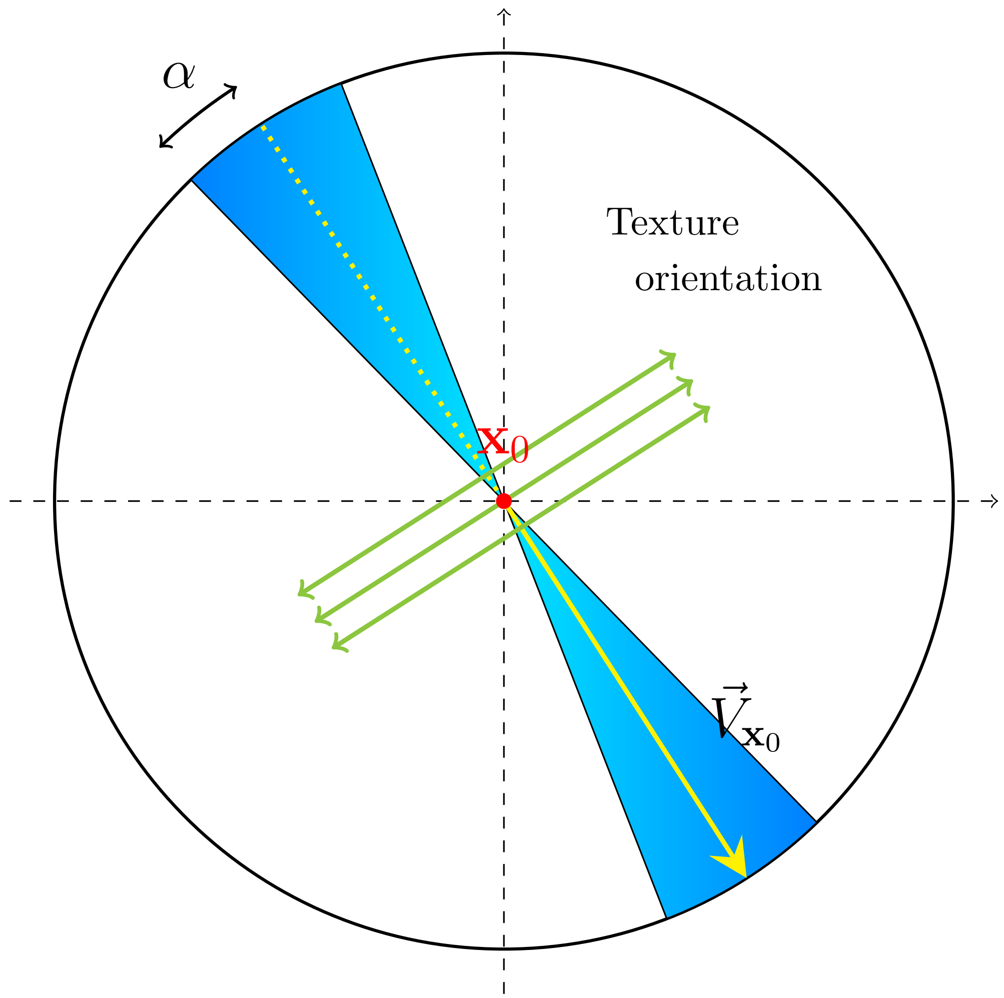

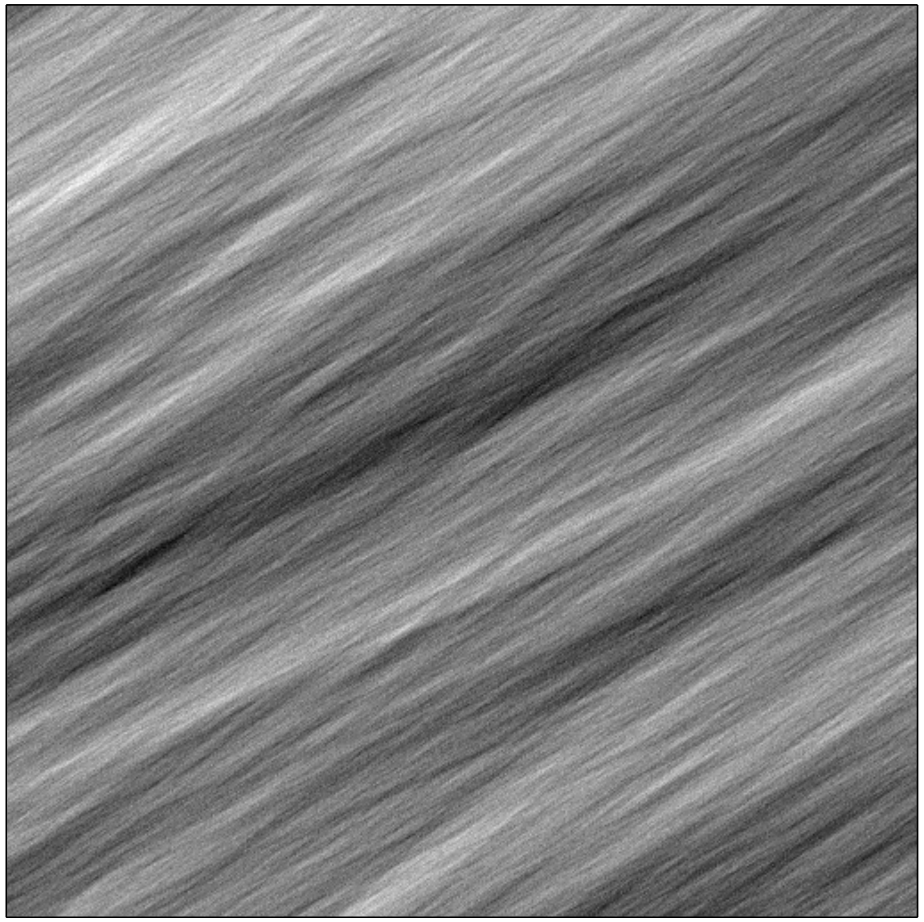

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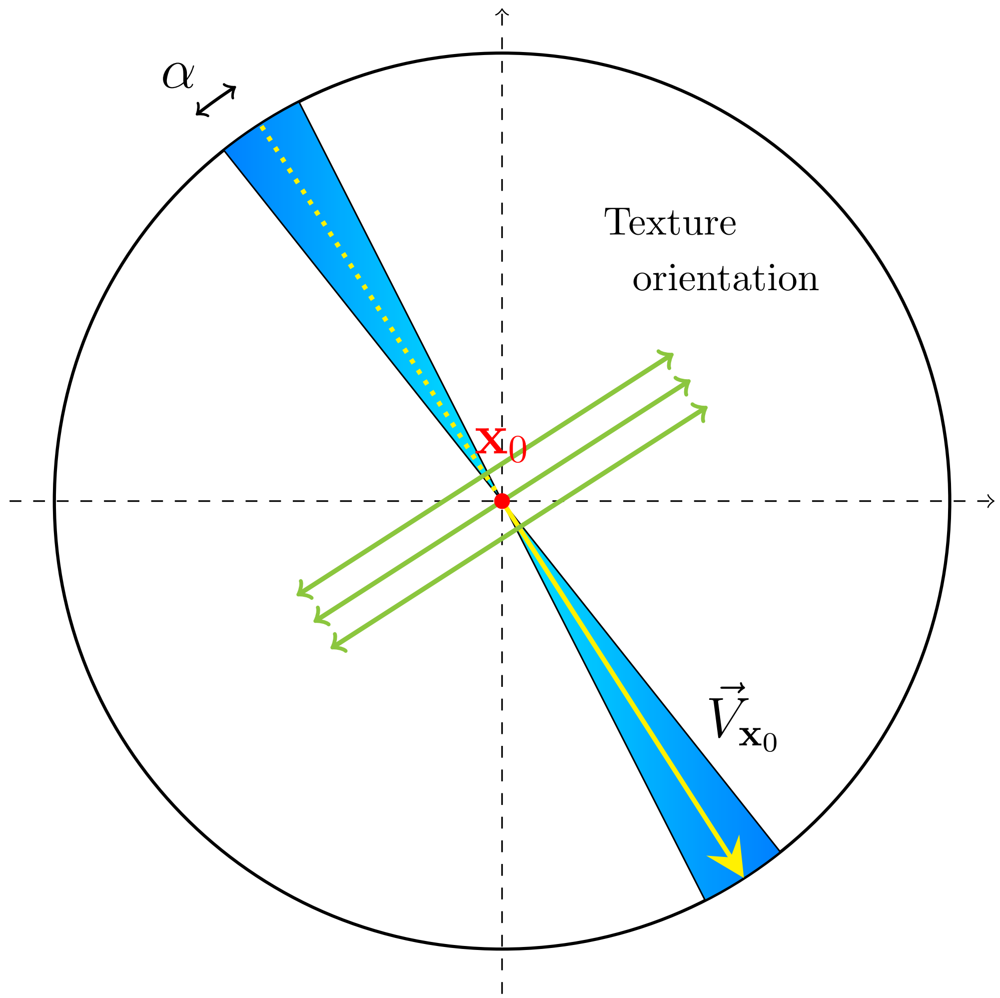

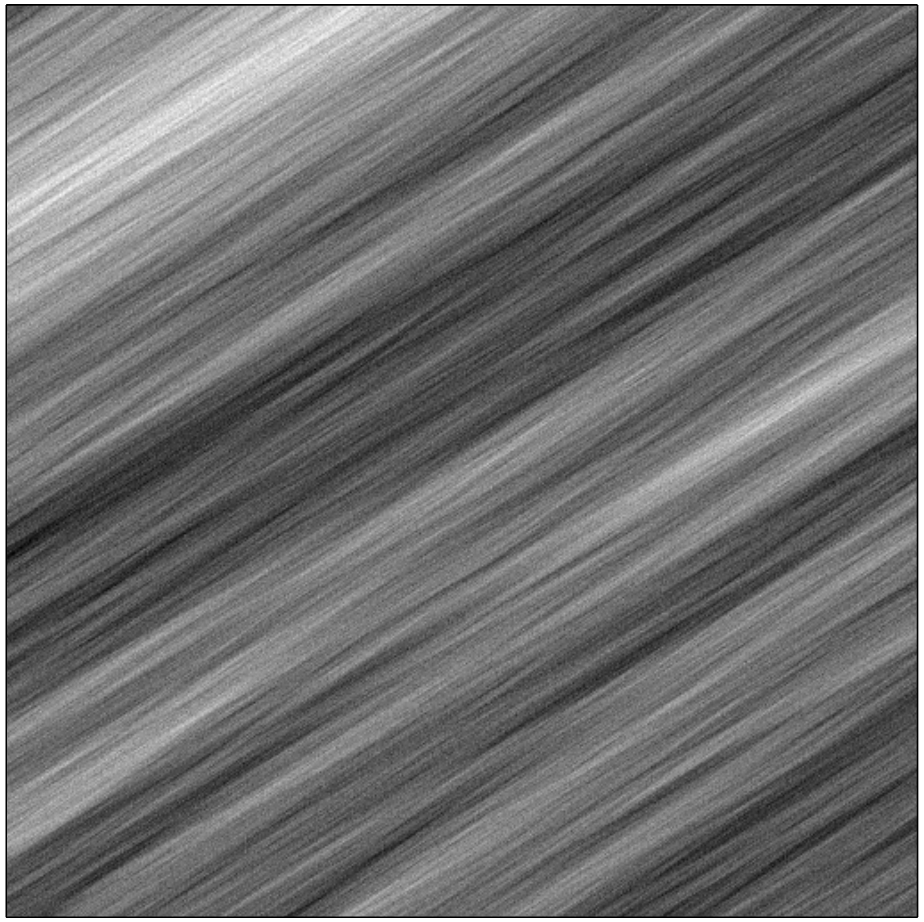

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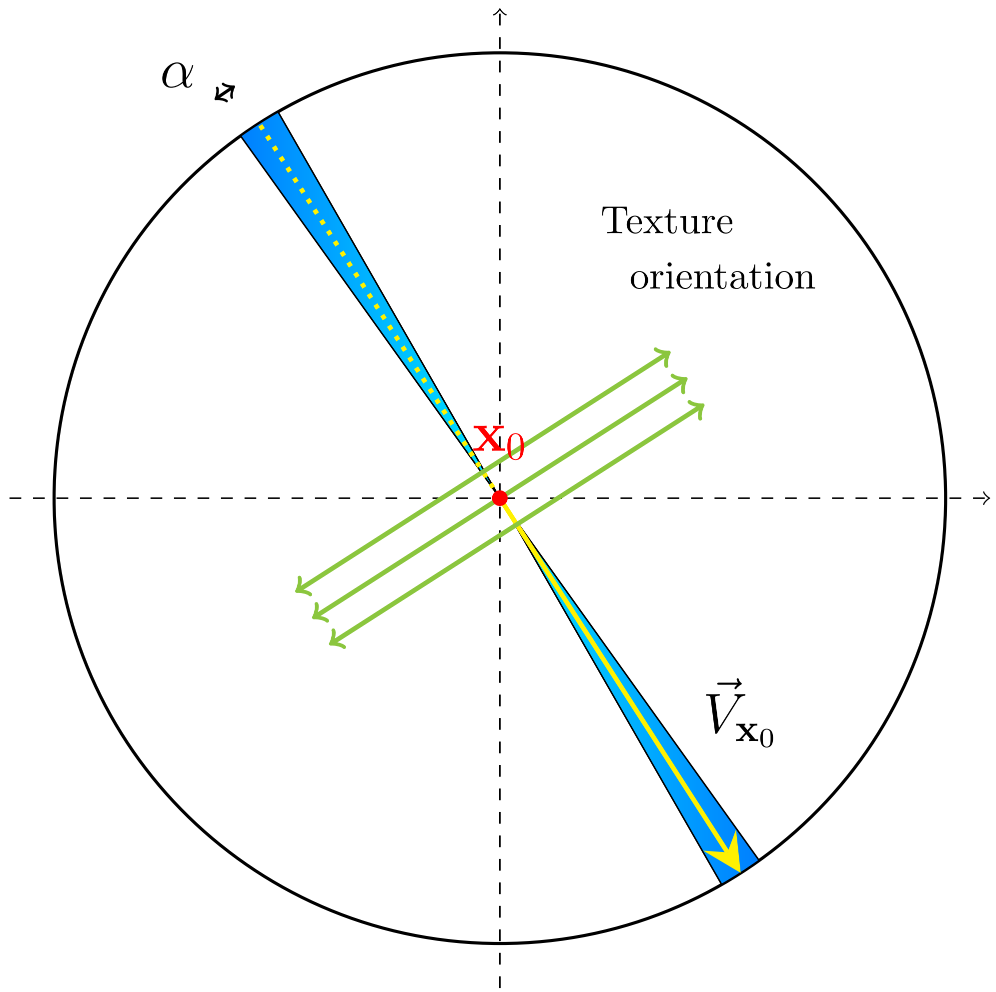

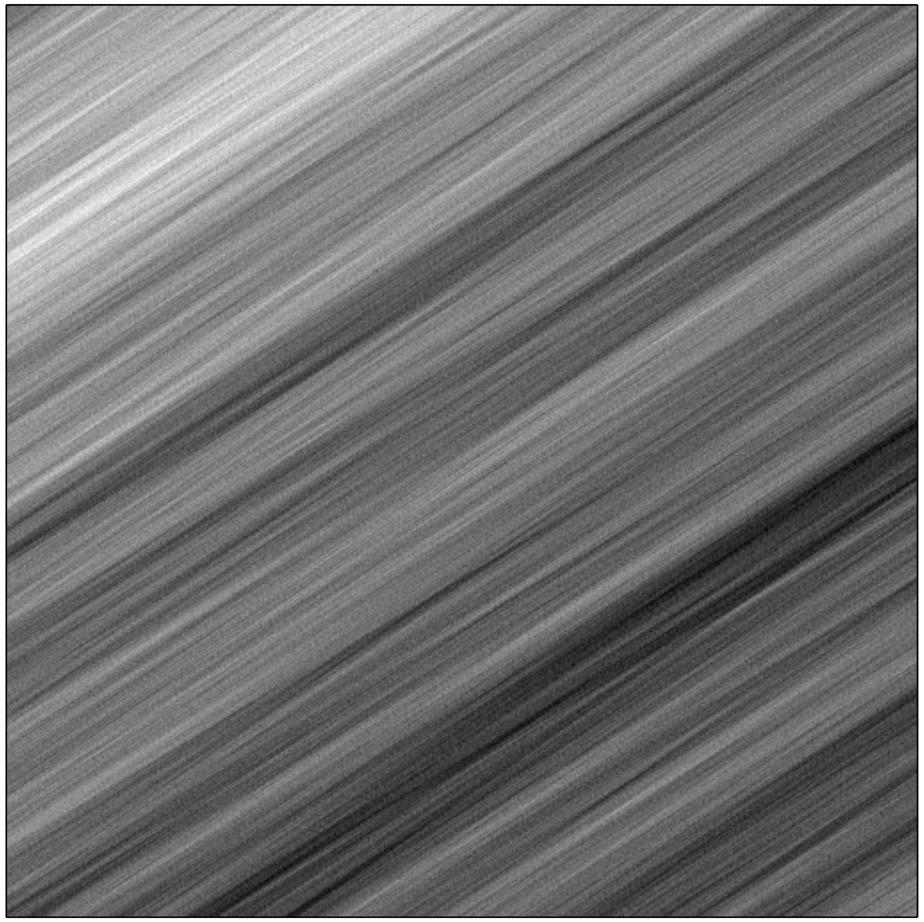

$$\alpha = 0.05$$

Tangent field

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Tangent field.

For a random field X locally asymptotically self-similar of order H,

$$\frac{X(\mathbf{x}_0 + \rho\mathbf{h}) - X(\mathbf{x}_0)}{\rho^H} \xrightarrow[\rho \to 0]{\mathcal{L}} Y_{\mathbf{x}_0}$$

$\longrightarrow$ $Y_{\mathbf{x}_0}$: tangent field of X at point $\mathbf{x}_0 \in \mathbb{R}^2$

[Benassi,1997]
[Falconer,2002]

Taylor's expansion $\Longleftrightarrow$ Tangent field

Deterministic case — *Stochastic case*

Tangent field

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Theorem. The LAFBF $B^H_{\alpha_0,\alpha}$ admits for tangent field $Y_{\mathbf{x}_0}$:

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$\longrightarrow$ $Y_{\mathbf{x}_0}$ *elementary field* with global orientation $\alpha_0(\mathbf{x}_0)$

Tangent field

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constant

⟶ $Y_{\mathbf{x}_0}$ *elementary field* with global orientation $\alpha_0(\mathbf{x}_0)$

Tangent field

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constant

$\longrightarrow$ $Y_{\mathbf{x}_0}$ *elementary field* with global orientation $\alpha_0(\mathbf{x}_0)$

$\Rightarrow$ $B^{H}_{\alpha_0,\alpha}(\mathbf{x}_0) \approx Y_{\mathbf{x}_0}(x = \mathbf{x}_0)$

Simulation of tangent fields

Continuous formulation. Variogram of $Y_{\mathbf{x}_0}$: [Bierme,Richard,Moisan,2012]

$$
\begin{aligned}
v_{Y_{\mathbf{x}_0}}(\mathbf{x}) &= \frac{1}{2}\int_{\mathbb{R}^2} |e^{i\mathbf{x}\cdot\xi} - 1|^2 f(\mathbf{x}_0,\xi)d\xi \\
&= \frac{1}{2}\gamma(H)\int_{-\pi/2}^{\pi/2} c_{\alpha_0,\alpha}(\mathbf{x}_0,\theta)\ |\mathbf{x}\cdot\mathbf{u}(\theta)|^{2H}d\theta \\
&= \int_{-\pi/2}^{\pi/2} \tilde{v}_\theta(\mathbf{x}\cdot\mathbf{u}(\theta))d\theta
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in polar coordinates

$$\tilde{v}_\theta = \frac{1}{2}\gamma(H) c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta)|\cdot|^{2H}$$

$$\mathbf{u}(\theta) = (\cos\theta, \sin\theta)$$

$$\gamma(H) = \frac{\pi}{H\Gamma(2H)\sin(H\pi)}$$

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in polar coordinates

variogram of a fractional brownian motion (FBM) of order H

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\end{aligned}
$$

in polar coordinates

variogram of a fractional brownian motion (FBM) of order H

$Y_{\mathbf{x}_0}$

$=$

Infinite sum of independant rotating FBM of order H

$$\tilde{v}_\theta = \frac{1}{2}\gamma(H) c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta) |\cdot|^{2H}$$

$$\mathbf{u}(\theta) = (\cos\theta, \sin\theta)$$

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Discrete formulation.

[Bierme,Richard,Moisan,2012]

- $(\theta_i)_{1\leqslant i\leqslant n}$ are n bands orientations and $\lambda_i = \theta_{i+1} - \theta_i$
- The **turning band field** is defined as

$$Y_{\mathbf{x}_0}{}^{[n]}(\mathbf{x}) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}(\mathbf{x}_0, \theta_i)} B_i^H(\mathbf{x} \cdot \mathbf{u}(\theta_i))$$

- B_i^H are n independent FBM of order H
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Simulation of tangent fields

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not equispaced ✗

Simulation of tangent fields

Simulation along particular bands. [Bierme,Richard,Moisan,2012]

- Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0,1]^2$ with $r = 2^k - 1, k \in \mathbb{N}^\star$
- Choose (θ_i) such that $\tan\theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leqslant \epsilon$
- Then $B_i^H(\mathbf{x}\cdot\mathbf{u}(\theta_i))$ becomes

$$\left\{B_i^H\left(\frac{k_1}{r}\cos\theta_i + \frac{k_2}{r}\sin\theta_i\right); 0 \leqslant k_1, k_2 \leqslant r\right\} \overset{\mathcal{L}}{=} \left(\frac{\cos\theta_i}{rq_i}\right)^H \{B_i^H(k_1 q_i + k_2 p_i); 0 \leqslant k_1, k_2 \leqslant r\}$$

Simulation of tangent fields

Simulation along particular bands.

[Bierme,Richard,Moisan,2012]

- Discrete grid $r^{-1}\mathbb{Z}^2 \cap [0,1]^2$ with $r = 2^k - 1, k \in \mathbb{N}^\star$
- Choose (θ_i) such that $\tan\theta_i = \frac{p_i}{q_i}$ and $\max_i \lambda_i \leqslant \epsilon$
- Then $B_i^H(\mathbf{x}\cdot\mathbf{u}(\theta_i))$ becomes

Dynamic programming

$$\left\{ B_i^H\left(\frac{k_1}{r}\cos\theta_i + \frac{k_2}{r}\sin\theta_i\right); 0 \leqslant k_1, k_2 \leqslant r \right\} \stackrel{\mathcal{L}}{=} \left(\frac{\cos\theta_i}{rq_i}\right)^H \{B_i^H(k_1 q_i + k_2 p_i); 0 \leqslant k_1, k_2 \leqslant r\}$$

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equipaced ✓

Simulation of LAFBF

using tangent fields

[Polisano et al.,2014]

Algorithm. For each pixel $\mathbf{x}_0 = (k_1, k_2) \in [\![0, r]\!]^2$

$$B^{H}_{\alpha_0,\alpha}((k_1, k_2)) = \gamma(H)^{\frac{1}{2}} \sum_{i=1}^{n} \sqrt{\lambda_i c_{\alpha_0,\alpha}((k_1, k_2), \theta_i)} \left(\frac{\cos\theta_i}{rq_i}\right)^{H} B_i^H(k_1 q_i + k_2 p_i)$$

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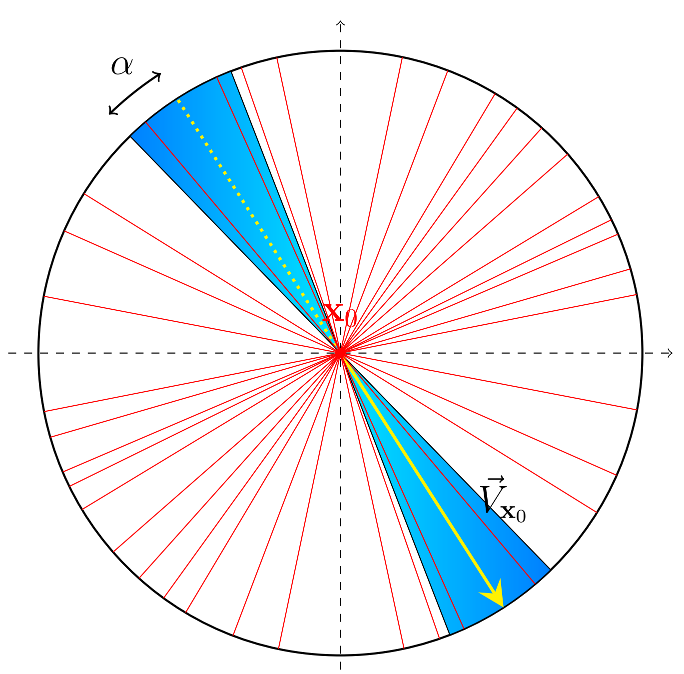

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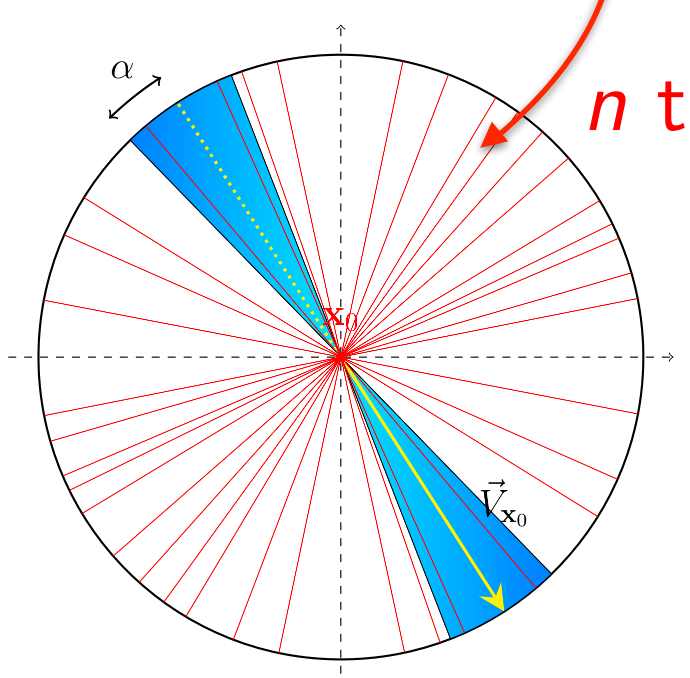
n turning bands θ_i



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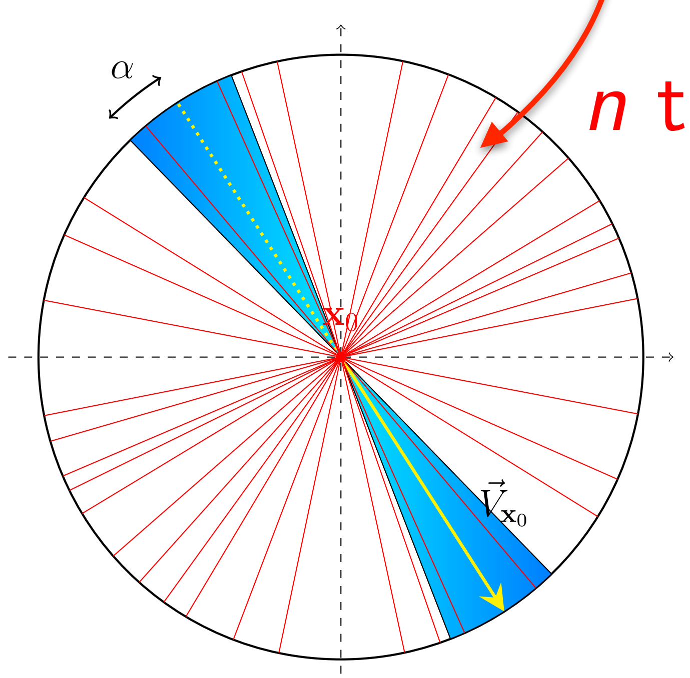

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$$\mathbb{1}_{[-\alpha,\alpha]}(\theta_i - \alpha_0((k_1,k_2))) \neq 0 \Leftrightarrow |\theta_i - \alpha_0((k_1,k_2))| \leqslant \alpha$$

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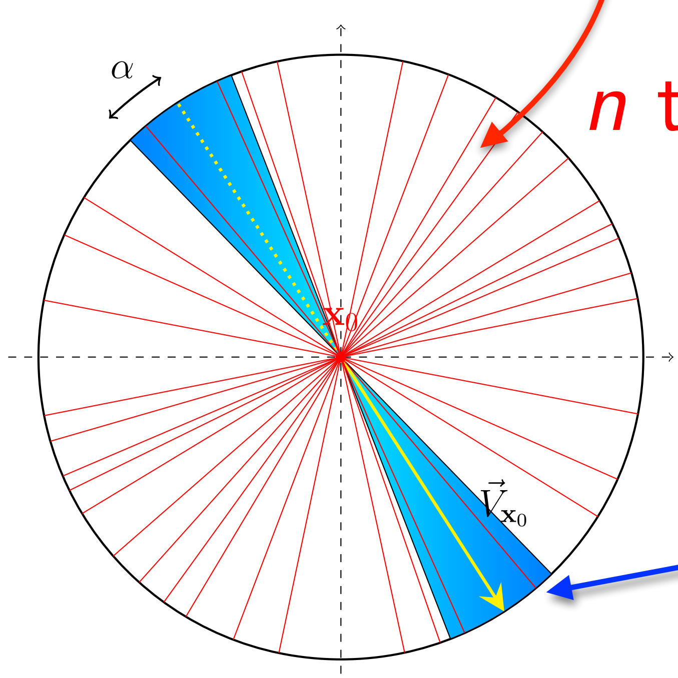

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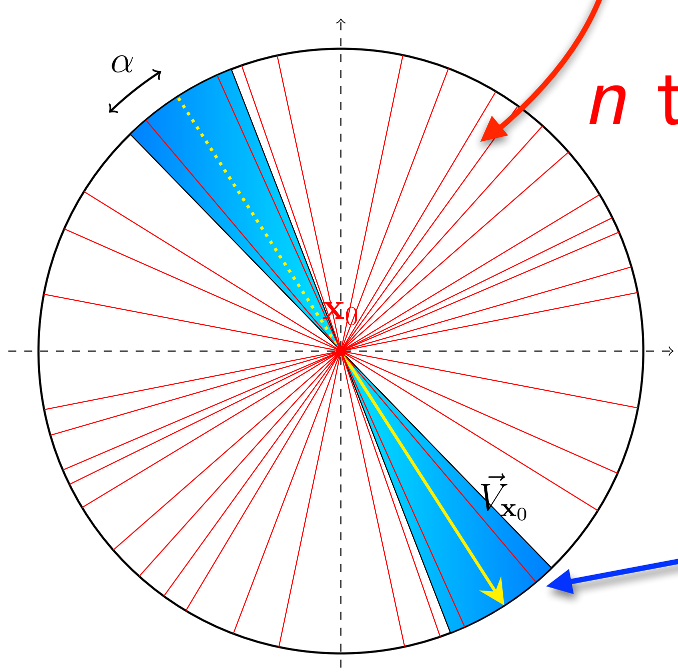

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few bands in the cone

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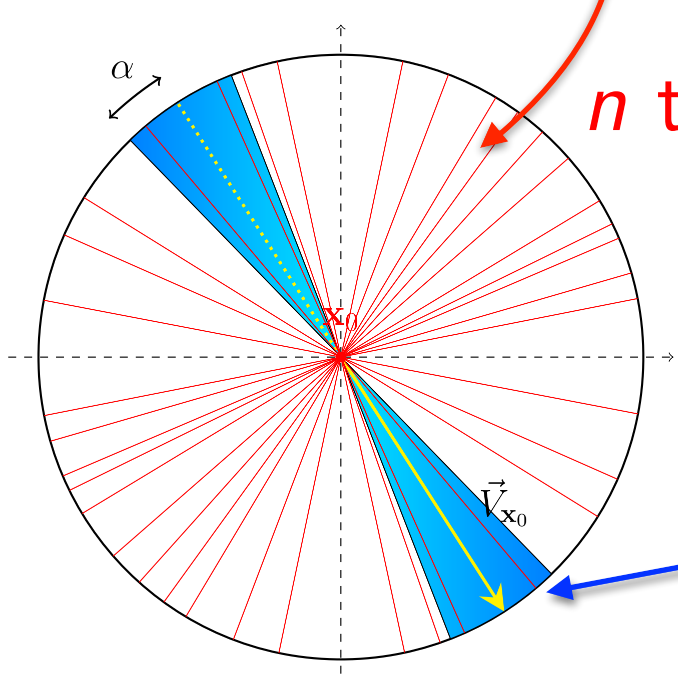

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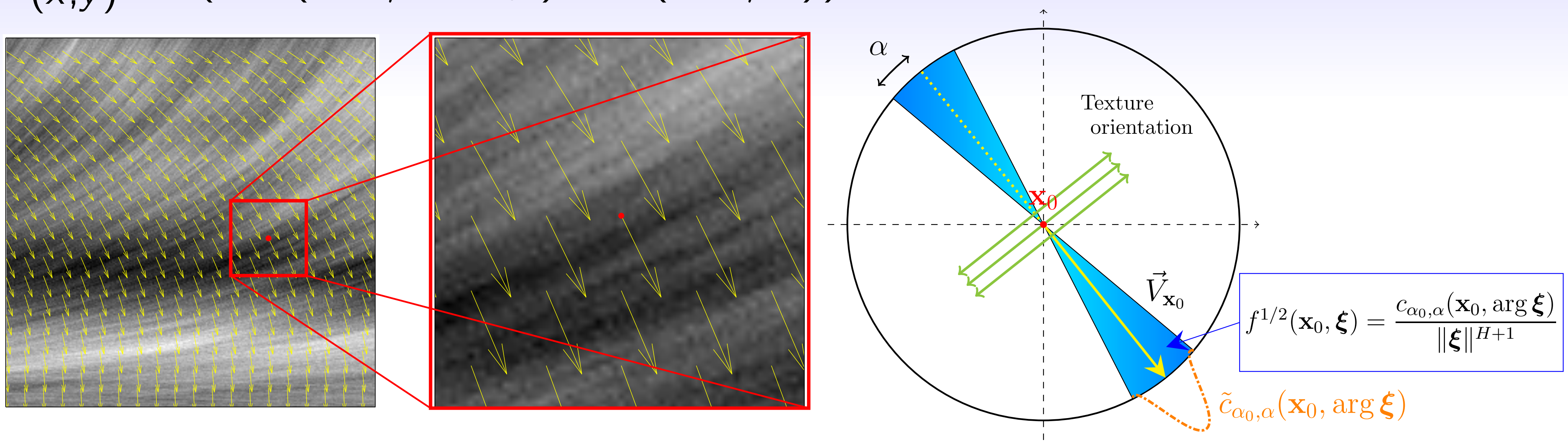
few bands in the cone

Complexity $O(r^2 \log n)$

Numerical experiments

$$\vec{V}^1_{(x,y)} = (\cos(-\pi/2 + y), \sin(-\pi/2))$$



(a) (b) (c)

Parameters

$r = 255 \quad H = 0.2$

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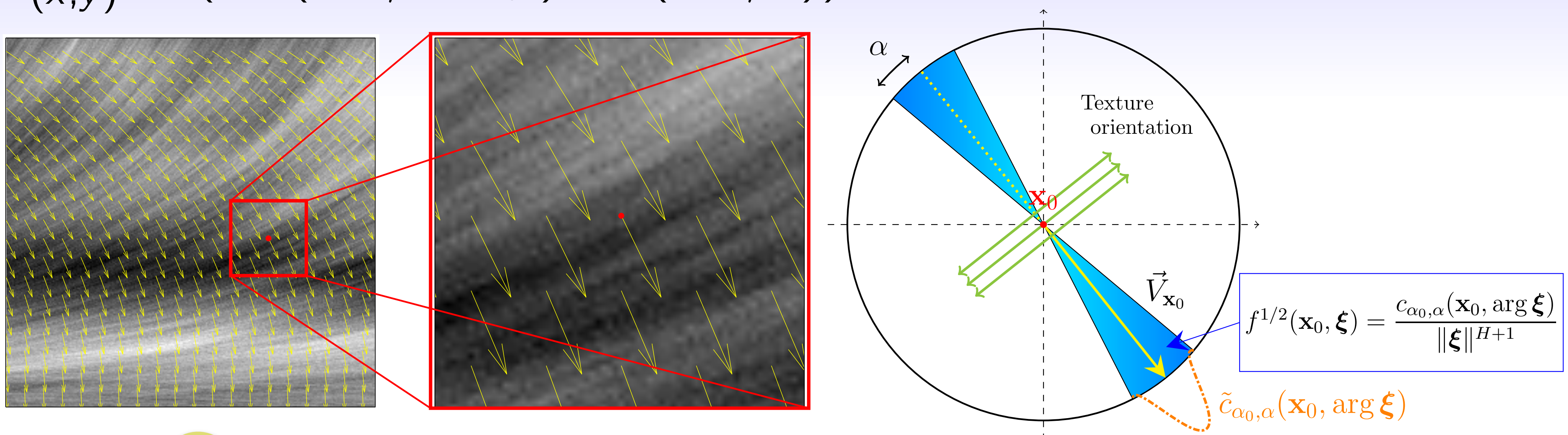

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Texture with prescribed local orientation at each point $\mathbf{x}_0$ given by a vector field

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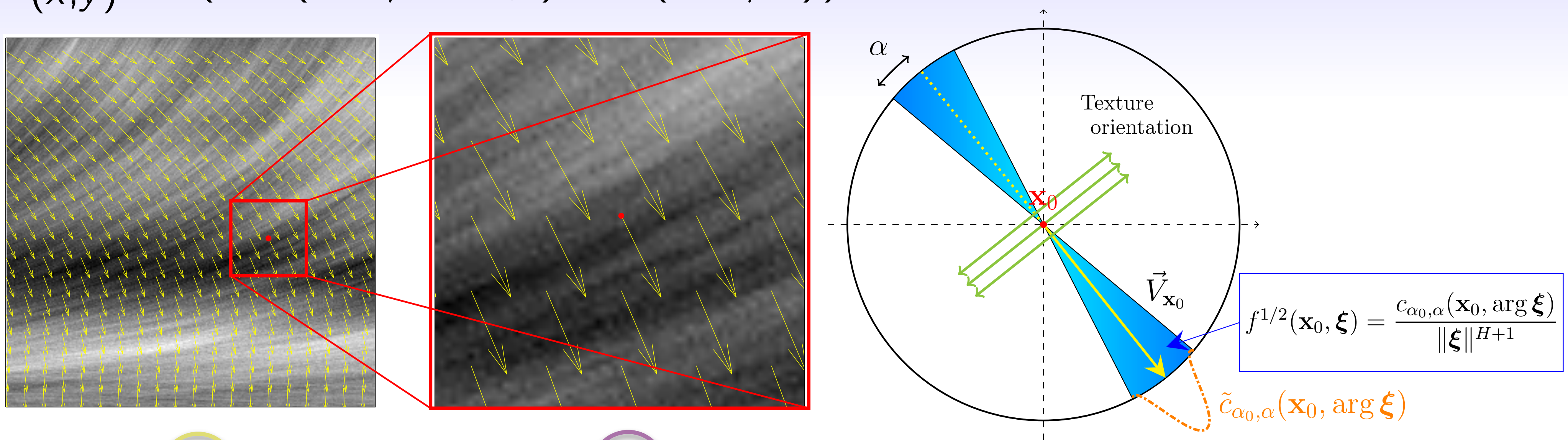

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A zoom around a point $\mathbf{x}_0$ shows that locally a LAFBF behaves as an elementary field

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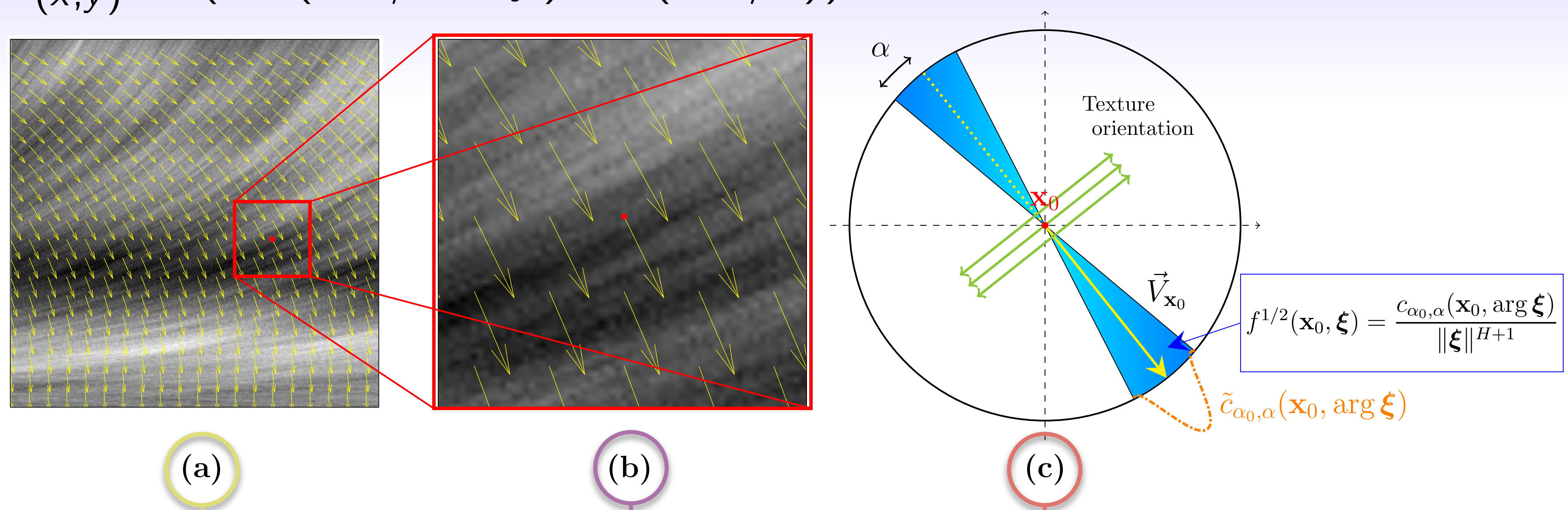

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Regularized version of the anisotropy function

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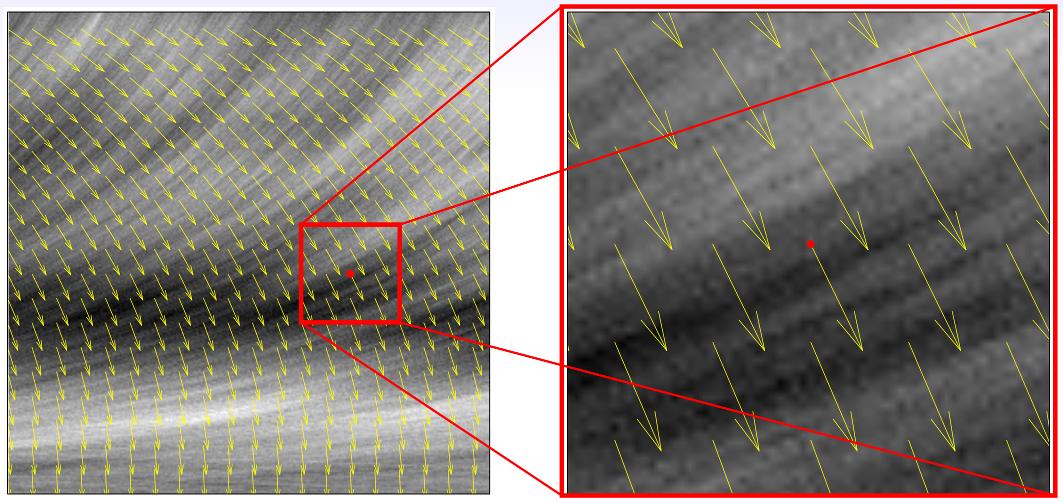

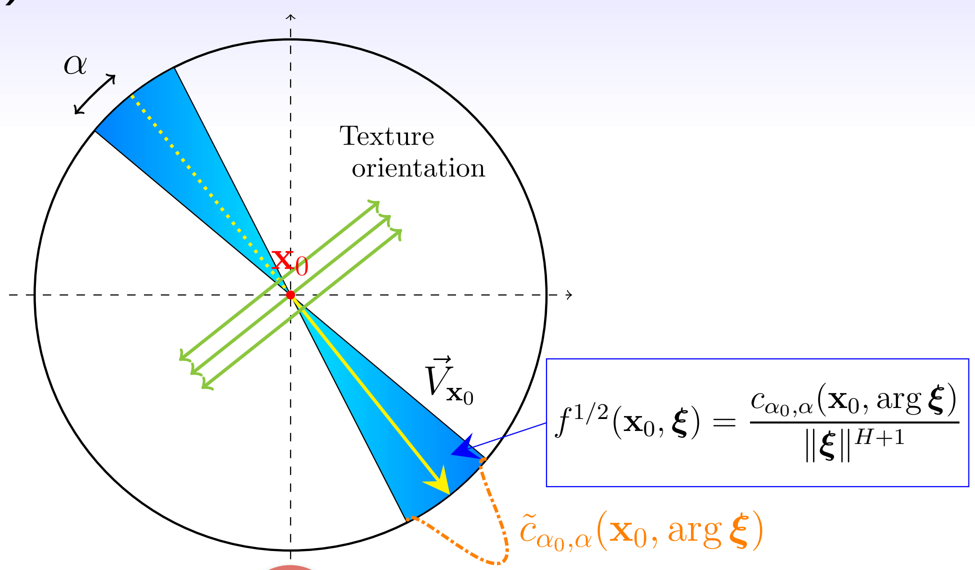

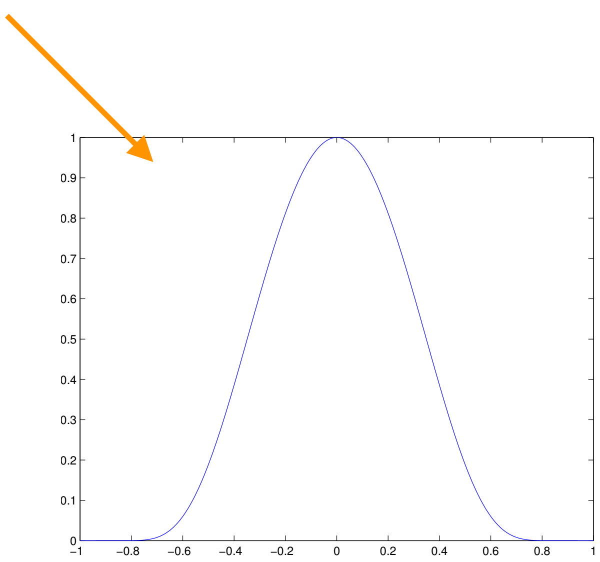

(a) Texture with prescribed local orientation at each point $\mathbf{x}_0$ given by a vector field

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(c) Regularized version of the anisotropy function

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$\vec{\mathbf{V}}^2_{(x,y)} = (\cos(\cos(36xy)), \sin(\cos(36xy)))$

$\vec{\mathbf{V}}^3_{(x,y)} = \nabla F(x, y)$

$F(x, y) = (4x - 2)e^{-(4x-2)^2-(4y-2)^2}$

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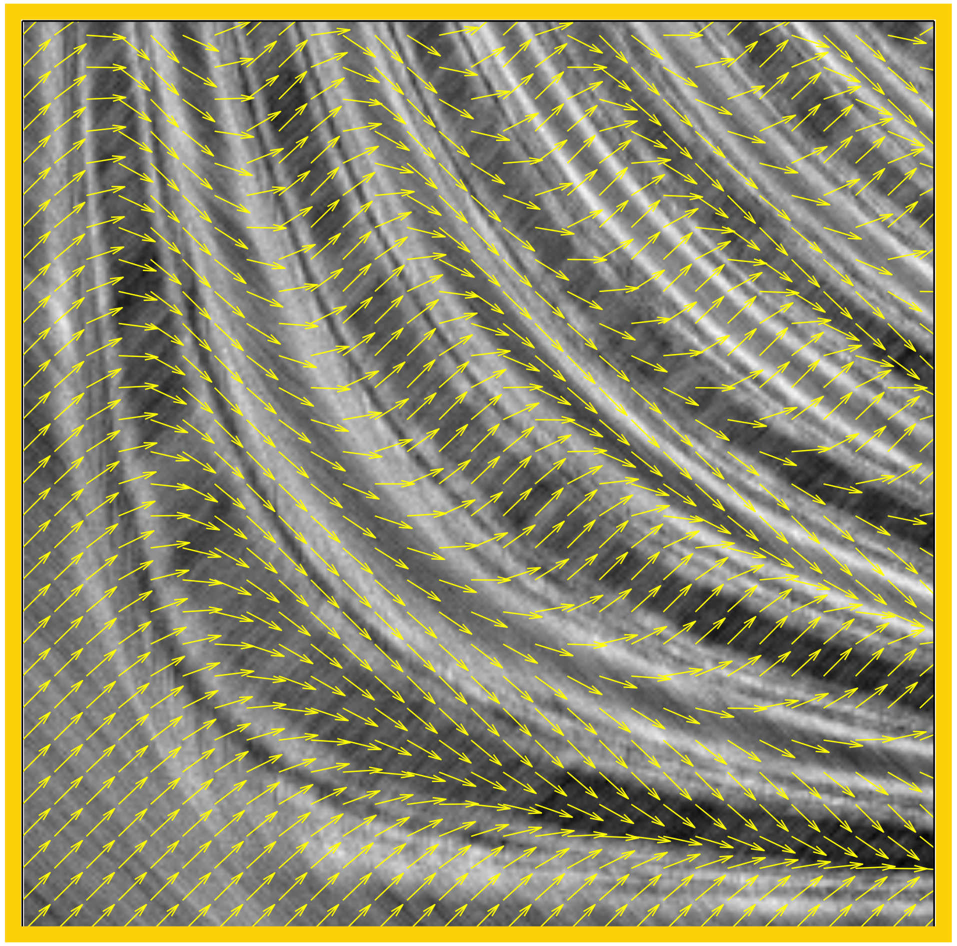

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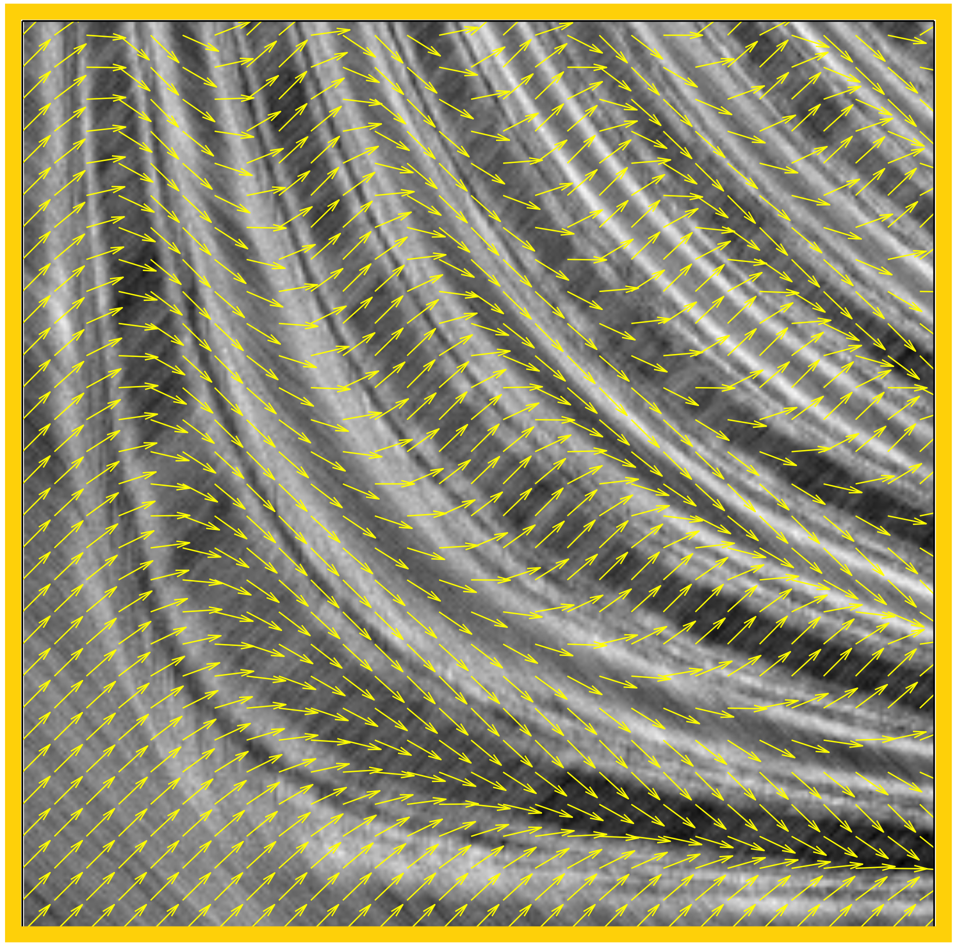

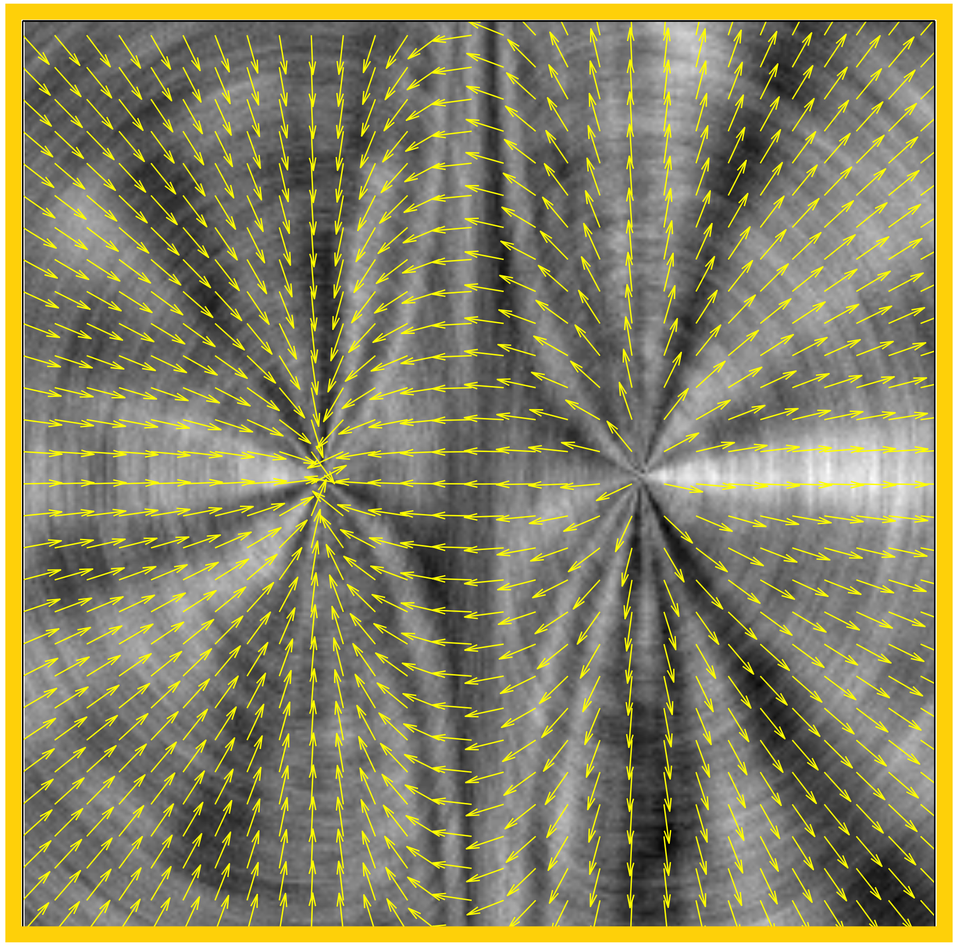

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Numerical experiments

H=0.2 H=0.5

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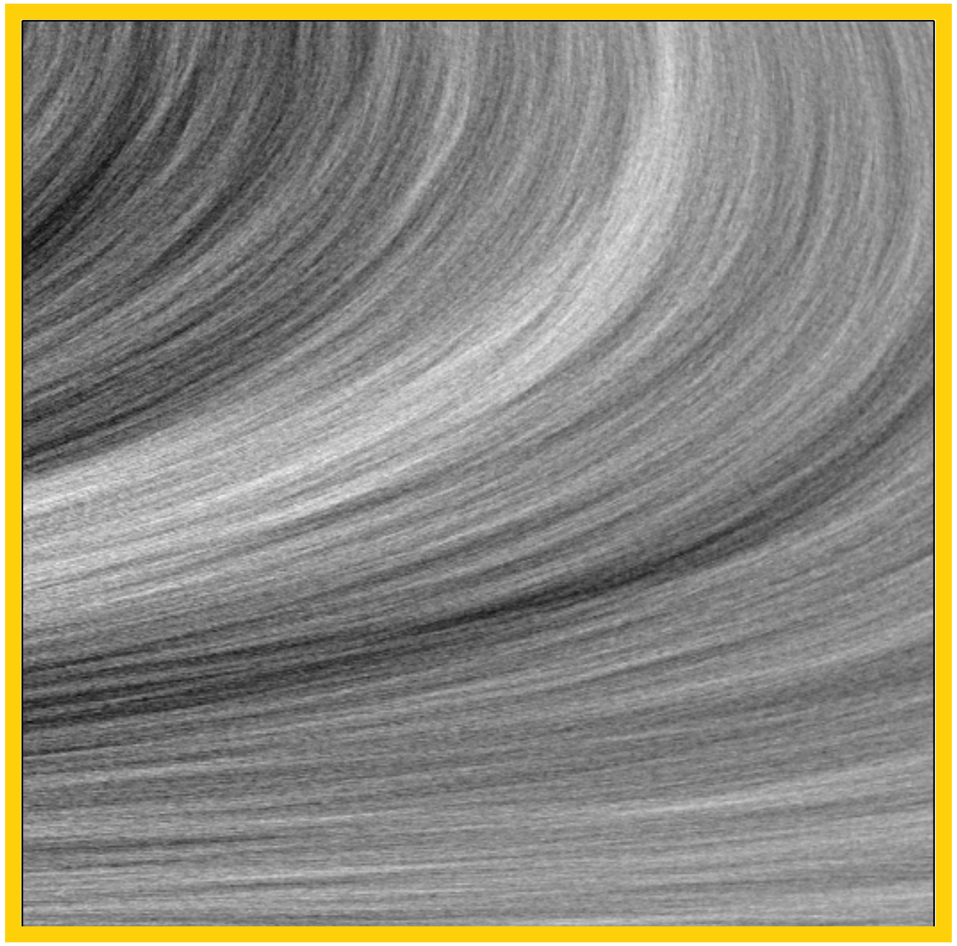

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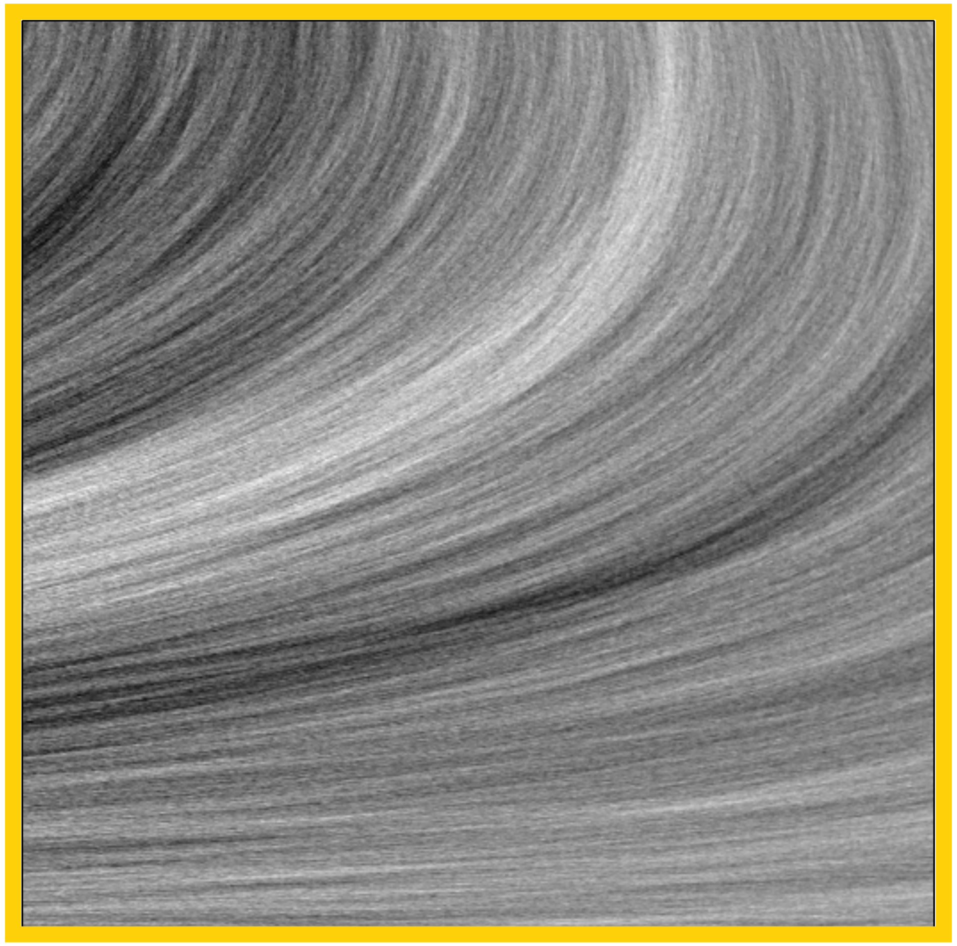
Numerical experiments

H=0.2



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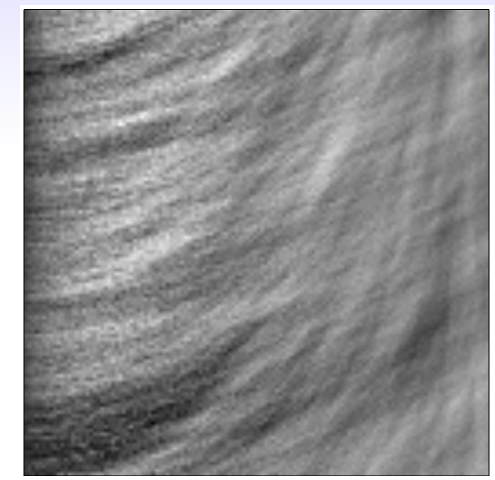

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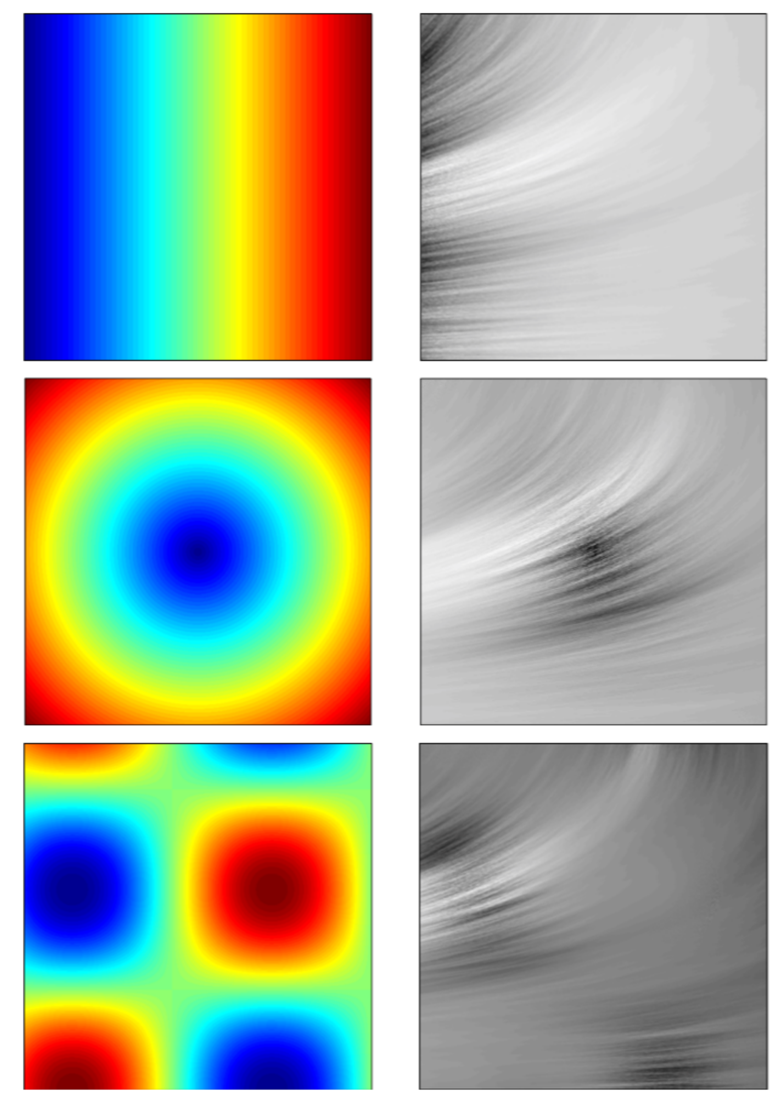

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- K.J. Falconer, "Tangent fields and the local structure of random fields," *Journal of Theoretical Probability*, vol. 15, no. 3, pp. 731–750, 2002.

Questions ?

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Thank you for your attention.

- **Dynamic programming.** The choice of the bands orientation $(\theta_i)_{1 \leqslant i \leqslant n}$ is governed by the computational cost of the B_i^H 's within dynamic programming.
  - Let the error ϵ fixed. Taking $N = \lceil \frac{1}{\tan \epsilon} \rceil$ consider the following set:

$$\mathcal{V}_N = \left\{ (p,q) \in \mathbb{Z}^2 / -N \leqslant p \leqslant N, 1 \leqslant q \leqslant N, p \wedge q = 1, -\frac{\pi}{2} < \arctan\left(\frac{p}{q}\right) < \frac{\pi}{2} \right\}$$

  - The aim is to find n pairs in the set $\mathcal{V}_N$ which minimize the following global cost:

$$C(\Theta) = \sum_{k=1}^{s} C(r(|p_{i_k}| + q_{i_k}))$$

where $C(\ell)$ is the cost of one FBM B_i^H in $O(n \log n)$, under the constraint $\max_i (\theta_{i+1} - \theta_i) \leqslant \epsilon$